Elementary Graphing, Distance Formula, & Graphs of Circles

Graphing in the plane consists of plotting coordinate pairs. In this section we will see how the distance formula is used.

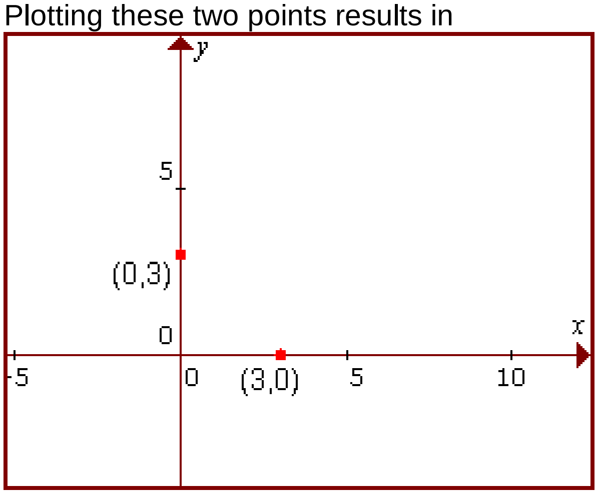
Graphing Points
When graphing a point (x, y), the first coordinate defines the location on the horizontal axis and the second coordinate defines the location on the vertical axis. So a point (3,-5) would have a horizontal coordinate of 3 and a vertical coordinate of –5.

Elementary Graphing of Equations
If you are graphing equations for the first time and you have no specialized method to use, you can graph many equations by doing the following:

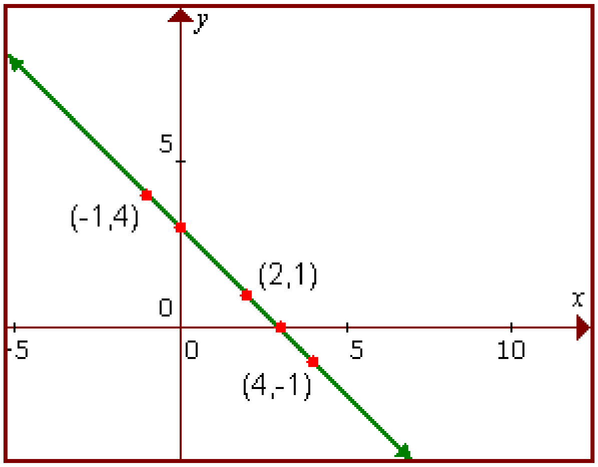
1. Find and plot all y-intercepts. To find these let x = 0 and solve for y. Then find and plot all x-intercepts. To find these, let y=0 and solve for all values of x.
2. Find and plot more points on in between the intercepts you plotted and on each side of the intercepts you plotted.
3. Draw a smooth curve through your plotted points from left to right.

Example: Graph 2y + 2x = 6

If x=0, 2y + 2(0) = 6 which simplifies to 2y=6, with y=3. The x-intercept is (3,0).
If y=0, 2(0) + 2x = 6 which simplifies to 2x=6, with x=3. The y-intercept is (0,3).
Plotting these two points results in



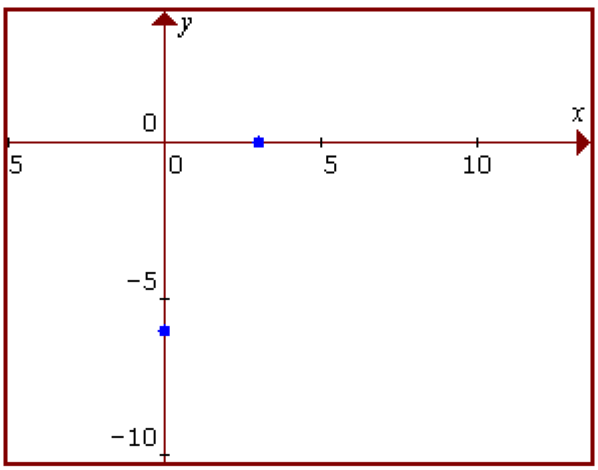
Now, find some more points. Plug in x=2, x=4, and x=-1 and find the corresponding y-values. For x=2 you get 2y + 4 = 6 with y=1, for x=4 you get 2y + 8 = 6 with y=-1, and for x=-1 you get 2y –2 = 6 with y=4. Pairing up your values results in the points (2,1), (4,-1), and (-1, 4). Plot these points, and then draw a smooth curve through – in this case we end up with a straight line as shown on the next page.



Example: Graph y = -2|x – 3|

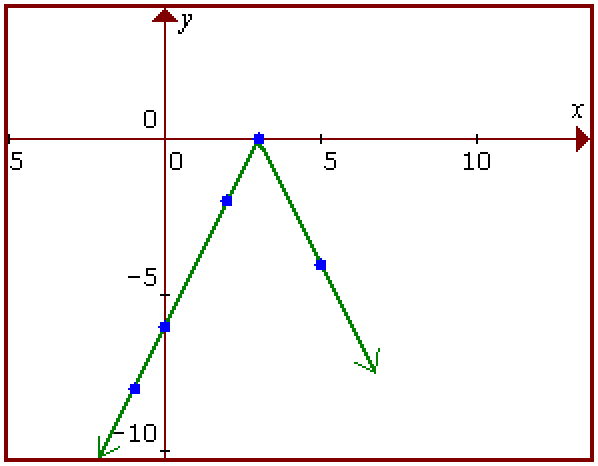
To find the y-intercept, we let x=0, resulting in y = -2|0 – 3| = -2|– 3|= -2(3) = -6. Note that we took absolute value of –3 here. So the y-intercept is (0, -6).

To find the x-intercept, we let y=0, resulting in 0 = -2|x – 3|.
Apply the Division Property of Equality to divide both sides by –2 to get 0 = |x– 3|. The two cases for this are really the same case. We solve 0 = x – 3 to get x = 3. So the x-intercept is (3,0). The points are plotted and shown below.



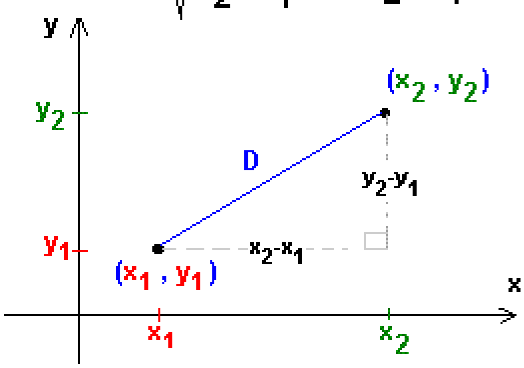
Now, you need to find some more points to see what this graph is doing. Some good values of x to use to find more points would be x=2, x=5, and x=-1 since these will give us points on each side of the intercepts.

Plugging x=2, x=5, and x=-1 into y = -2|x – 3| results in y=-2, y=-4, and y=-8, respectively, so the points are (2,-2), (5,-4), and (-1, -8). Plot these points and draw a curve through from left to right, as shown on the next page.



The Distance Formula
The distance formula states:

$$D = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$



where D is the distance between (x_1, y_1) and (x_2,y_2). As may be seen in the graph above, the distance from point to point really is the hypotenuse of a right triangle of sides length ($x_2 - x_1$) and ($y_2 - y_1$).

To use the Distance Formula, simply plug the values of (x_1, y_1) and (x_2,y_2) into the formula.

Example: Find the distance from (4,-1) and (-2,2).

Plug the values from (4, -1) and (-2, 2) into (where $x_1 = 4$, $y_1 = -1$, $x_2 = -2$, $y_2 = 2$)

$$D = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

To get

$$D = \sqrt{(2 - (-1))^2 + (-2 - 4)^2}$$

$$D = \sqrt{45}$$

or $3\sqrt{5}$ if you simplify the radical.

Equation of a Circle
A circle of radius r, centered at (h,k) is defined as all points (x, y) that are "r" units from the center (h,k). If we apply the distance formula to this relationship, where the distance from (h,k) to (x, y) is r, we get the formula

$$(x - h)^2 + (y - k)^2 = r^2$$

Example: If a circle has a center at (3,-1) with radius 2, what is its equation?
Here, h=3, k = -1, and r = 2. We substitute these values into the circle formula to get $(x – 3)^2 + (y – (-1))^2 = 2^2$ which simplifies to

$$(x – 3)^2 + (y + 1)^2 = 4$$

And here is the graph of this circle: