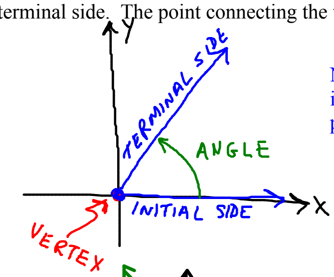
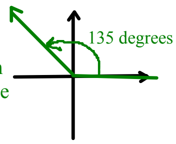
Welcome to Section 6.1 - Angles & Their Measures

An ANGLE is determined by rotating a ray about its endpoint as shown below. We call the starting position of the ray the initial side. We call the position after rotation the terminal side. The point connecting the two sides is called the vertex.

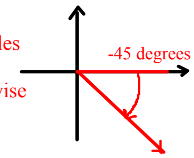


Note that we position the initial side on the x-axis and we position the vertex at the origin.

Positive angles are generated with a counterclockwise rotation.



Negative angles are generated with a clockwise rotation.

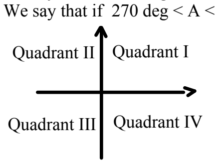


A note on notation. The capital letters A, B, C, D and so on will be used to denote angles because it is impossible to insert Greek characters "theta", "alpha", etc. Also, "deg" will be used in place of the degree symbol since that symbol is not contained in the font available.

DEFINITIONS

Let "A" represent an angle measure.

We say that if 0 deg < A < 90 deg, the angle is in the FIRST quadrant.
We say that if 90 deg < A < 180 deg, the angle is in the SECOND quadrant.
We say that if 180 deg < A < 270 deg, the angle is in the THIRD quadrant.
We say that if 270 deg < A < 360 deg, the angle is in the FOURTH quadrant.



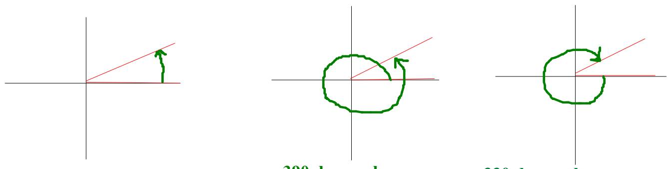
Note that the angles 0 deg, 90 deg, 180 deg, & 270 deg are known as "quadrant angles".

"Acute" angles are defined as angles with degree measure from 0 to 90 degrees.

"Obtuse" angles are defined as angles with degree measure from 90 to 180 degrees.

Coterminal Angles

Two angles are "coterminal" if they have the same terminal ray. An example would be the angles 30 deg, 390 deg, and -330 deg shown below. These are all coterminal because the terminal ray is in the same location for each.



30 deg angle | **390 deg angle (360 deg + 30 deg)** | **-330 deg angle (This is 330 deg rotation in clockwise direction)**

Example

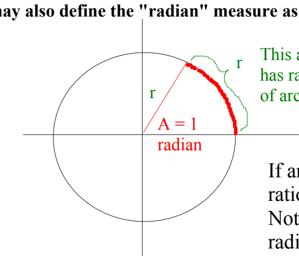
Name 4 angles that are coterminal with 60 degrees.

Answer: (60 + 360) deg = 420 deg
(60 - 360) deg = - 300 deg
(60 + 720) deg = 780 deg
(60 - 720) deg = -660 deg

Radian Measure - An angle may be measured in terms of "Radians" rather than degrees.

1 Radian is the measure of a central angle "A" that intercepts an arc "s" equal to the radius "r" of the circle. An example of an angle with measure 1 radian is shown below.

We may also define the "radian" measure as the ratio of the arc length to the radius.



This arc has a length of "r" units. By definition, this angle has radian measure of 1 radian. Also note that the ratio of arc length to radius is r/r = 1

If an angle is "A" radians, this means that the ratio of the arc length to the radius is "A". Note that in this ratio, the units cancel and radians is a dimensionless unit.

Since one full revolution about the circle covers a distance equal to the circumference of (2 pi)(r), and this is obtained by forming a 360 degree angle formed on a circle of radius r, (2pi)r/r = 2pi and then 360 degrees = 2 pi radians, and 180 deg = 1 pi radians. It is helpful to remember the following:

pi radians = 180 deg

Note: "pi" is used to represent 3.14159 . . . due to the lack of the Greek symbol for pi.

Example: Convert the angles 30 deg, -45 deg, 390 deg, and 112 deg to radians.

Remember that 180 deg = pi radians. Thus, multiply each of these by the following conversion factor: $\frac{\pi \text{ RADIANS}}{180 \text{ DEG}}$

Note that the degree units cancel.

$$\frac{30^{\circ}}{1} \times \frac{\pi \text{ RADIANS}}{180^{\circ}} = \frac{30\pi}{180} \text{ RADIANS} = \frac{\pi}{6}$$

Note that we may leave off the "radians" units or we may include them if we want.

$$\frac{-45^{\circ}}{1} \times \frac{\pi}{180^{\circ}} = \frac{-45\pi}{180} = -\frac{\pi}{4}$$

$$\frac{390^{\circ}}{1} \times \frac{\pi}{180^{\circ}} = \frac{390\pi}{180} = \frac{39\pi}{18} = \frac{13\pi}{6}$$

Note that we may let pi = 3.14159 . .

$$\frac{112^{\circ}}{1} \times \frac{\pi}{180^{\circ}} = \frac{112\pi}{180} = \frac{56\pi}{90} \approx 1.955 \text{ RADIANS}$$

Example: Convert the radian angles $\frac{\pi}{3}, \frac{7\pi}{8}, 3$ into degrees.

Note that in each case, we multiply by the conversion factor 180deg/pi.

$$\frac{\pi}{3} \times \frac{180^{\circ}}{\pi} = \frac{180^{\circ}}{3} = 60^{\circ}$$

Remember that pi = 180 deg.
So pi/3 is 1/3 of 180 deg = 60 deg.

$$\frac{7\pi}{8} \times \frac{180^{\circ}}{\pi} = 157.5^{\circ}$$

$$\frac{3}{1} \times \frac{180^{\circ}}{\pi} = \frac{540}{\pi}^{\circ} \approx 171.9^{\circ}$$

Pi radians = 180 deg. Since 3 is slightly less than pi, we would expect 3 radians to be equal to slightly less than 180 deg.

Note that most scientific calculators have a key to convert between degrees & radians. Consult your users manual for the key(s) required.

DMS measurement of angles

In addition to radians and degrees, another way to measure angles is the Degree, Measurement, Second system which is abbreviated DMS. In this system

D = degrees
M = minutes, where 60 minutes = 1 degree 22 minutes would be written as 22'
S = seconds, where 60 seconds = 1 minute 11 seconds would be written as 11"
Note that 3600 seconds would = 1 degree.

For example, 45 deg, 13 minutes, 32 seconds is written as $45^\circ\ 13'\ 32''$

Example: Convert $45^\circ\ 13'\ 32''$ into just plain degrees.

Multiply 13 minutes by the ratio 1 deg/60 min to get 13/60 deg.

Multiply 11 seconds by the ratio 1 deg/3600 seconds to get 11/3600 deg.

Thuss 45deg, 13' 32" would = 45 deg + 13/60 deg + 11/3600 deg = 45.2197 deg (rounded)

Example: Convert 32.5 deg into DMS.
0.5 deg would be equal to 1/2 deg which would be equal to 1/2 of 60 or 30minutes.

Thus, 32.5 deg = 32 deg, 30 minutes.

Arc Length

Arc length "s" is the length along the edge of a circle containing an angle of "A" radians.

Since "A" radians is defined as the ratio of arc length "s" divided by radius "r", or

A = s/r.

Solving for s results in s = (r)A, where angle "A" is measured in radians.

Example: How far does the end of a 12" second hand travel when going from 12 to 4?

When the second hand travels from the 12 to the 4, it is going 4/12 =1/3 of a revolution or 2pi/3 radians. Thus, our angle "A" in radians is 2pi/3.

The distance that the end of the second hand travels is the arc length and is equal to
s = (r) A = (12") (2pi/3) = 8pi " which is about 25.1 ".

Angular Speed

Angular speed is defined as the rate of change of the angle "A" per time t. Since s = (r)A, and "A" = s/r, angular speed = A/t = s/(rt) where "t" is the time for angle "A" to be formed.

Example: A car is moving at a speed of 30 mph, and the diameter of its wheels is 2.5 ft.
A) How fast in rpm are the wheels rotating?
B) What is the angular speed of the wheels in radians per minute?

First note that 30 mph means that each 1 hour = 60 minutes, 30 miles are traveled.

30 miles multiplied by (5280 ft/mile) = 15840 feet. Thus, the speed in feet per minute is
15840 ft/60 min = 264 ft/min.

Each rotation of the wheel accounts for 2pi(r) = 2(pi)(1.25) ft = 7.854 ft (rounded).

Note that the radius is 1/2 of the diameter of 2.5 ft = 1.25 ft.

Dividing 264 ft by 7.854 ft results in 33.61 rotations in each minute. Since revolutions per minute are the same as rotations per minute, the rpm of the wheels are 33.61 rpm.

To find the angular speed, you only need to realize that for each rotation, 2pi radians are travelled.

Thus, multiply $\frac{33.61 \text{ ROTATIONS}}{1 \text{ MIN}} \times \frac{2\pi \text{ RADIANS}}{1 \text{ ROTATION}}$

The angular speed is 211.2 radians per minute.

Do the Exercises assigned for Section 6.1. Read the text also!