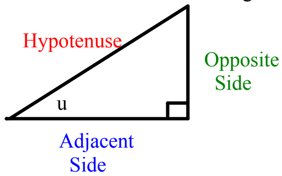
Welcome to Section 6.2 - Right Triangle Trigonometry

Given any right triangle, the ratio of any two side lengths of the triangle are defined in terms of trigonometric functions.

Let "u" = one of the acute angles of a right triangle. We then define the following:



Note: We abbreviate sine of angle u as sin u, cosine (u) is written cos u, tangent (u) is written tan u.

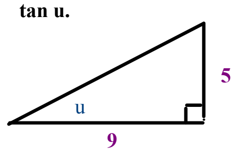
$$\text{Sine (u)} = \frac{\text{Opposite Side}}{\text{Hypotenuse}}$$

$$\text{Cosine (u)} = \frac{\text{Adjacent Side}}{\text{Hypotenuse}}$$

$$\text{Tangent (u)} = \frac{\text{Opposite Side}}{\text{Adjacent Side}}$$

We only need to remember Sin u = O/H, Cos u = A/H, Tan u = O/A with the acronym "SOHCAHTOA" where O=Opposite Side, A = Adjacent Side, H = Hypotenuse.

Example: Given the triangle below with side lengths given, find sin u, cos u, & tan u.



By the Pythagorean Theorem, the hypotenuse is found with the formula a^2 + b^2 = c^2, where c = hypotenuse and a and b are the shorter sides.

$$5^2 + 9^2 = (\text{HYPOTENUSE})^2$$

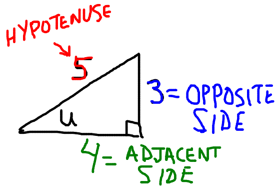
$$H = \text{HYPOTENUSE} = \sqrt{5^2+9^2} = \sqrt{106}$$

By definition, sin u = O/H = 5/sqrt(106)
cos u = A/H = 9/sqrt(106)
tan u = O/A = 5/9

Note: "O" = Opposite Side Length = 5
"A" = Adjacent Side Length = 9

Example: Sketch a right triangle with an acute angle u such that sin u = 3/5. Find the 3rd side and then find cos u and tan u.

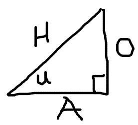
Since sin u = O/H, O = 3, H = 5. Using the Pythagorean Theorem to find the value of the 3rd side results in A = sqrt(5^2 - 3^2) = sqrt(16) = 4.
A sketch of the triangle is shown on the next page.



Cos u = A/H = 4/5
Tan u = O/A = 3/4

Other Trigonometric Functions

Given a right triangle with acute angle "u", we define the following:



Cosecant u = H/O
Secant u = H/A
Cotangent u = A/O

Note that we abbreviate
Cosecant u = Csc u
Secant u = Sec u
Cotangent u = Cot u

Note also that
Csc u = 1/Sin u
Sec u = 1/Cos u
Cot u = 1/Tan u

Finding the Trig Functions of a Given Angle With a Calculator

If you are given an acute angle "u" of a right triangle, you may find the values of sin u, cos u, and tan u by using a scientific calculator. With the exception of some special case angles, it is actually necessary to use your calculator.

Example: Given a right triangle with acute angle u = 22 deg, find sin u, cos u, tan u, csc u, sec u, and cot u.

In older scientific calculators, you enter the angle "22", and then you press one of the keys labelled "sin", "cos", "tan".

In newer scientific calculators, you press the key "sin", "cos", "tan", and then enter the angle "22" and a right parenthesis).

Note: Since you are finding the function of an angle in "degrees", your calculator must be in "degrees mode" rather than "radians mode". To switch from one mode to another, you usually press a key called "mode", however each make of calculator is different so you may have to consult your users manual or ask someone for help on this.

The answers you should get, rounded to 2 decimal places are sin 22 = 0.37, cos 22 = 0.93, tan 22 = 0.40 . To find csc 22, sec 22, cot 22, remember that csc u = 1/sin u, sec u = 1/cos u, cot u = 1/tan u and take the reciprocal of each of the answers for sin, cos, and tan to get csc 22 = 2.67, sec 22 = 1.08, cot 22 = 2.48 .

Example: Find sin u, if u = pi/5 RADIANS by using your calculator.

You must first switch the mode on your calculator to RADIANS MODE.

On an older scientific calculator, you would then enter the following in order to find sin (pi/5).

$\pi \quad \div \quad 5 \quad = \quad \text{SIN}$

Your answer should be 0.5877852 . . .
Did you switch to radians mode?

On a newer scientific calculator, you would enter the following:

$\text{SIN} (\pi \div 5) =$

Note that you need to close off the parenthesis.

Special Case Angles to Remember

There are two special-case triangles that you will encounter many times in this course and courses beyond this; the 45-45-90 triangle, and the 30-60-90 triangle.

See if you can construct a 45-45-90 triangle with shorter sides of length 1 and 1, and then find the hypotenuse and calculate the values of sine, cosine, tangent, secant, cosecant, and cotangent of the 45 degree angles. SEE NEXT PAGE

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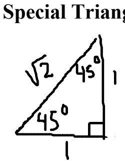
Special Triangle - 45-45-90



The hypotenuse, by the Pythagorean Theorem is equal to

$\sqrt{1^2+1^2} = \sqrt{2}$.

The 6 trig functions are calculated as shown here:

$\text{SIN } 45° = \frac{O}{H} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}, \quad \text{CSC } 45° = \frac{1}{\text{SIN } 45°} = \sqrt{2}$

$\text{COS } 45° = \frac{A}{H} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}, \quad \text{SEC } 45° = \frac{1}{\text{COS } 45°} = \sqrt{2}$

$\text{TAN } 45° = \frac{O}{A} = \frac{1}{1} = 1, \quad \text{COT } 45° = \frac{1}{\text{TAN } 45°} = 1$

Note: $\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

Can you try to construct a 30-60-90 triangle by bisecting an equilateral triangle (60-60-60) with side lengths 1,1, &1 into two equal 30-60-90 triangles and then calculate the trig functions of the angles 30 degrees and 60 degrees? This is how we can derive the values of these trig functions. See Next Page . . .

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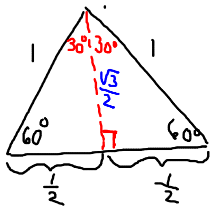
Special Triangle - 30-60-90



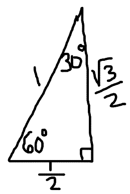
If we split the equilateral triangle in two equal halves, we obtain two 30-60-90 right triangles with side lengths 1, 1/2, and sqrt(3)/ 2. The side of sqrt(3)/2 is obtained with the Pythagorean Theorem in the following way:

$$h^2 + \left(\frac{1}{2}\right)^2 = 1$$

$$h^2 = 1 - \left(\frac{1}{2}\right)^2 = 1 - \frac{1}{4} = \frac{3}{4}$$

$$h = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$

This means that EVERY 30-60-90 triangle will have sides in the ratio shown here:



The trig functions are obtained with SOHCATOA:

$$\text{SIN } 30° = \frac{\left(\frac{1}{2}\right)}{1} = \frac{1}{2}$$

$$\text{COS } 30° = \frac{\left(\frac{\sqrt{3}}{2}\right)}{1} = \frac{\sqrt{3}}{2}$$

$$\text{TAN } 30° = \frac{\left(\frac{1}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\text{SIN } 60° = \frac{\left(\frac{\sqrt{3}}{2}\right)}{1} = \frac{\sqrt{3}}{2}$$

$$\text{COS } 60° = \frac{\left(\frac{1}{2}\right)}{1} = \frac{1}{2}$$

$$\text{TAN } 60° = \frac{\frac{\sqrt{3}}{2}}{\left(\frac{1}{2}\right)} = \sqrt{3}$$

Radian-form angles & the special case 45-45-90 and 30-60-90 triangles

$$\text{SIN } 45° = \text{SIN } \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\text{COS } 45° = \text{COS } \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\text{TAN } 45° = \text{TAN } \frac{\pi}{4} = 1$$

$$\text{SIN } 30° = \text{SIN } \frac{\pi}{6} = \frac{1}{2}$$

$$\text{COS } 30° = \text{COS } \frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

$$\text{TAN } 30° = \text{TAN } \frac{\pi}{6} = \frac{\sqrt{3}}{3}$$

$$\text{SIN } 60° = \text{SIN } \frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\text{COS } 60° = \text{COS } \frac{\pi}{3} = \frac{1}{2}$$

$$\text{TAN } 60° = \text{TAN } \frac{\pi}{3} = \sqrt{3}$$

Here, the special case angles are each converted to radians, and the value of each trig function is given.

The secant, cosecant, and cotangent functions could also be found by taking the reciprocal of each of these.

You will NEED to memorize these special cases or know how to derive each of these!

Some Fundamental Trigonometric Identities

Reciprocal and Quotient Identities

$\sin u = \dfrac{1}{\csc u}$ $\quad \cos u = \dfrac{1}{\sec u}$ $\quad \tan u = \dfrac{1}{\cot u}$

$\tan u = \dfrac{\sin u}{\cos u}$ $\quad \cot u = \dfrac{\cos u}{\sin u}$

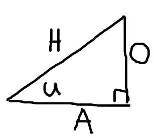
Pythagorean Identities

$\sin^2 u + \cos^2 u = 1$ Note that this is the same as $(\sin u)^2 + (\cos u)^2 = 1$

$1 + \tan^2 u = \sec^2 u$ Note that this is the same as $1 + (\tan u)^2 = (\sec u)^2$

$1 + \cot^2 u = \csc^2 u$ Note that this is the same as $1 + (\cot u)^2 = (\csc u)^2$

Can you show why the 1st Pythagorean Identity, sin^2 u + cos^2 u = 1 is true?
SEE NEXT PAGE FOR DERIVATION.



$\sin u = \dfrac{O}{H} \qquad \cos u = \dfrac{A}{H}$

$\sin^2 u = \left(\dfrac{O}{H}\right)^2 = \dfrac{O^2}{H^2} \qquad \cos^2 u = \left(\dfrac{A}{H}\right)^2 = \dfrac{A^2}{H^2}$

$\sin^2 u + \cos^2 u = \dfrac{O^2}{H^2} + \dfrac{A^2}{H^2}$

By the Pythagorean Theorem, O^2 + A^2 = H^2. $\quad = \dfrac{O^2 + A^2}{H^2} = \dfrac{H^2}{H^2} = 1$

Using Trig Identities to Verify Identities

Example: Verify $\dfrac{\sin u}{\cos u} + \dfrac{\cos u}{\sin u} = \csc u \sec u$

by showing that the left side is equal to the right side and using trigonometric identities.

Your first step will be to combine the fractions on the left by getting common denominators. SEE NEXT PAGE.

We may write the fractions on the left with common denominators.

$$\frac{\sin u}{\cos u}\cdot\frac{\sin u}{\sin u}+\frac{\cos u}{\sin u}\cdot\frac{\cos u}{\cos u}$$

We had to multiply each fraction by appropriate factors to obtain the common denominator of (cos u)(sin u)

$$=\frac{\sin^2 u}{\sin u\cos u}+\frac{\cos^2 u}{\sin u\cos u}$$

$$=\frac{\sin^2 u+\cos^2 u}{\sin u\cos u}$$

We combine the two fractions together. Then note that the numerator is equal to 1 because of the Pythagorean Identity.

$$=\frac{1}{\sin u\cos u}$$

$$=\frac{1}{\sin u}\cdot\frac{1}{\cos u}$$

$$=\csc u\cdot\sec u$$

We split this fraction into a product of two fractions. Since 1/sin u = csc u, 1/cos u = sec u, we can substitute and finally show that the left side is equal to the right side.

Cofunction Identities

If you noticed in the 30-60-90 triangle, the sine of 30 degrees was equal to the cosine of 60 degrees. The sine and cosine functions are known to be "cofunctions" of each other. Other cofunction pairs are tangent/cotangent and cosecant/secant. The following identities always apply.

$\sin(90^\circ - u) = \cos u$ $\quad$ $\cos(90^\circ - u) = \sin u$

$\tan(90^\circ - u) = \cot u$ $\quad$ $\cot(90^\circ - u) = \tan u$

$\sec(90^\circ - u) = \csc u$ $\quad$ $\csc(90^\circ - u) = \sec u$

Remember that cofunctions of complementary angles are equal, where complementary angles are two angles that add to 90 degrees.

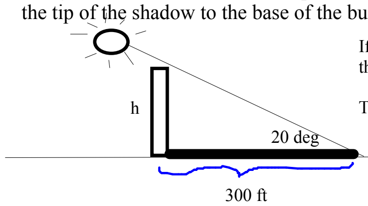
Note: All of the above identities should also be written with "pi" instead of 90 degrees if the angle "u" is in radians.

Example: If sin 20 degrees = 0.34202, what is cos 70 degrees?

Since sine and cosine are cofunctions and 20 deg & 70 deg are complementary angles, sin (20 deg) = cos(70 deg) = 0.34202

Application

The sun casts a shadow from a building such that the angle of elevation of the line of sight from the shadow to the top of the building is 20 degrees. The distance from the tip of the shadow to the base of the building is 300 feet. How tall is the building?



If we let "h" stand for the height of the building, then tan(20 deg) = O/A = h/300.

Then, h = 300(tan 20) = 109 ft (rounded)

Did you remember to use degrees mode?

Using the Inverse Sine, Cosine, and Tangent Keys on Your Calculator

On your calculator, you have inverse keys for which you input the value of the function and the calculator returns the ANGLE as an answer.

Example: Given sin u = 0.5, cos w = 0.1, tan v = 0.8, find the values of the angles u,w, v in both degrees and radians by using your calculator. SEE NEXT PAGE. . .

Given sin u = 0.5, enter the following:

0.5 , $\sin^{-1}$ OR $\sin^{-1}(0.5) =$

OLDER CALCULATOR NEWER CALCULATOR

Do this in degrees mode to get an answer of 30 degrees. Switch the mode to radians and you should get an answer of u = 0.5235987. . . radians.

In a similar manner, use the inverse cosine keys and inverse tangent keys to get

w = 84.26 degrees and w = 1.4706289. . . radians

v = 38.66 degrees and v = 0.6747409 . . . radians

Note that the inverse tangent and inverse cosine keys look like $\tan^{-1}$ & $\cos^{-1}$.

Do the exercises for Section 6.2 . Read the text!!!