Welcome to Section 6.4 - Graphs of Sine and Cosine Functions

In this section, you will learn the basic shapes of y = sin x and y = cos x.
You will then apply common function shift and translation rules to graph variations of y = sin x and y = cos x.
To begin with, calculate the values in the y-value column for y = sin x where x-values are given from 0 to 2pi at intervals of pi/4. Then, plot x vs. y.

x	y
0	?
pi/4	
pi/2	
3pi/4	
pi	0
5pi/4	
3pi/2	
7pi/4	
2pi	

y = sin x

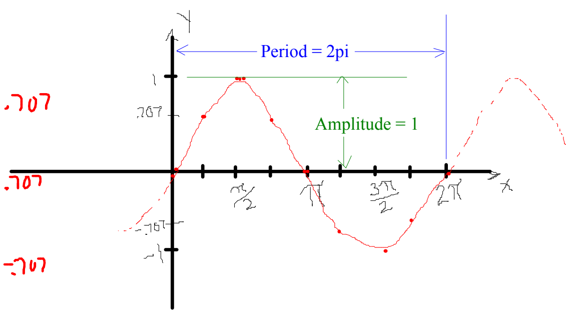
Calculate these y-values and plot the points on the x-y axis and draw a smooth curve through the points. Look for a repeating pattern that occurs.

How often does the graph repeat itself?

What are the maximum and minimum y-values that occur?

SEE NEXT PAGE . . .

x	y
0	0
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2} \approx .707$
$\frac{\pi}{2}$	1
$\frac{3\pi}{4}$	$\frac{\sqrt{2}}{2} \approx .707$
π	0
$\frac{5\pi}{4}$	$-\frac{\sqrt{2}}{2} \approx -.707$
$\frac{3\pi}{2}$	-1
$\frac{7\pi}{4}$	$-\frac{\sqrt{2}}{2}$
2π	0



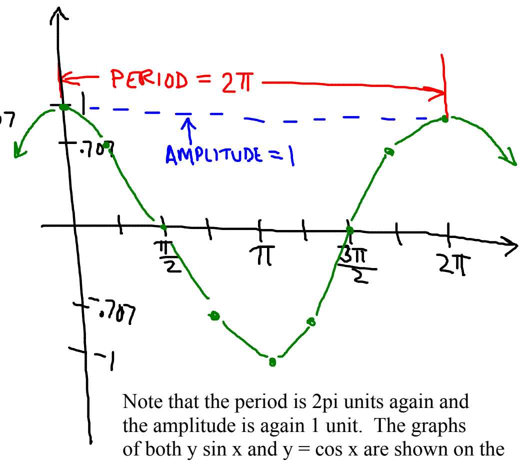
Notice that this graph repeats itself every 2pi units. We define this x-amount for the graph to repeat itself the "Period". This graph has a period of 2pi. The maximum y-value is 1 and the minimum y-value is -1.

We call the height from the center of this curve to the maximum value the "amplitude" of this function. Here, amplitude = 1.

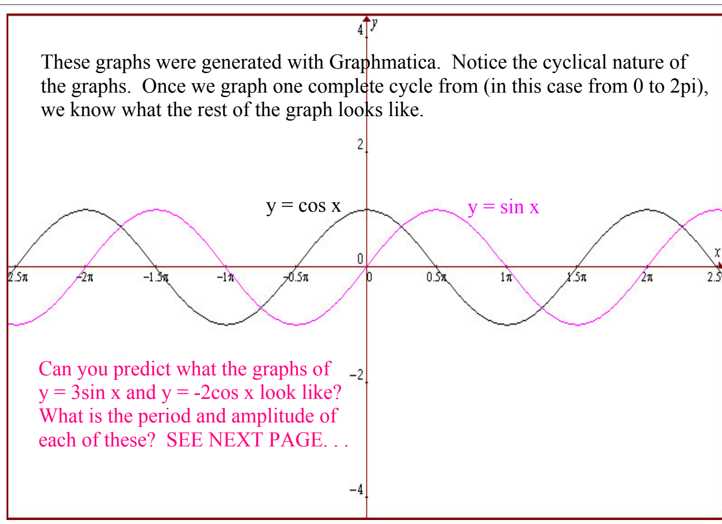
Use these same x-values, calculate y-values, and graph y = cos x. Then find the period and amplitude. SEE NEXT PAGE. . .

$y = \cos x$

x	y
0	1
$\pi/4$	$\frac{\sqrt{2}}{2} \approx .707$
$\pi/2$	0
$3\pi/4$	$-\frac{\sqrt{2}}{2}$
π	-1
$5\pi/4$	$\frac{-\sqrt{2}}{2}$
$3\pi/2$	0
$7\pi/4$	$\frac{\sqrt{2}}{2}$

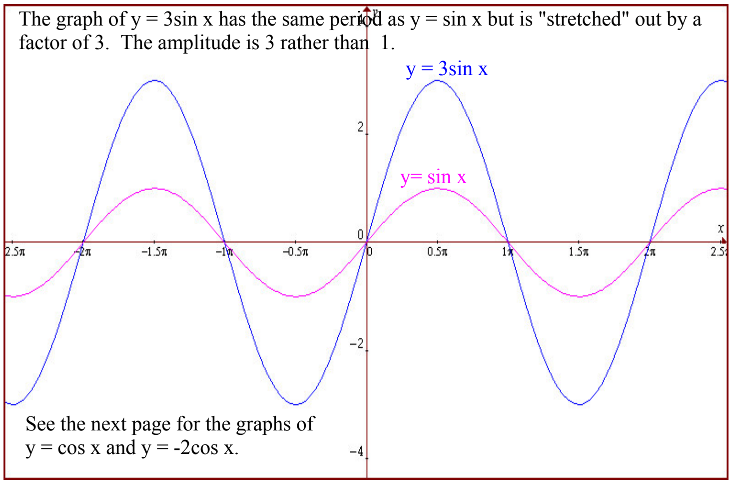


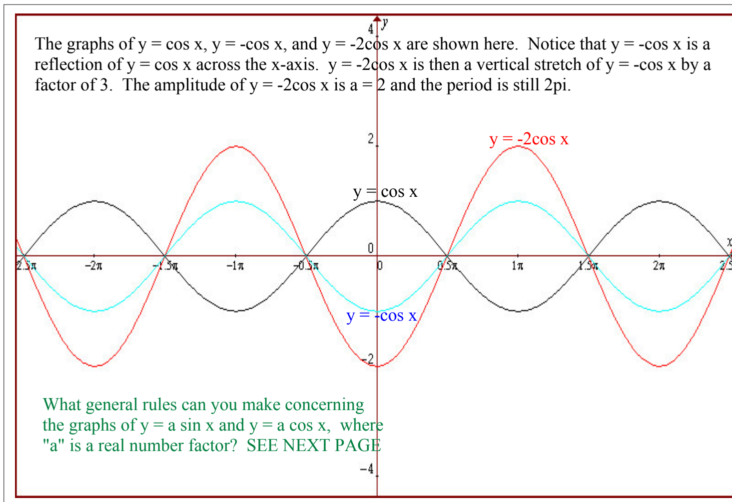
Note that the period is 2pi units again and the amplitude is again 1 unit. The graphs of both y sin x and y = cos x are shown on the next page.



These graphs were generated with Graphmatica. Notice the cyclical nature of the graphs. Once we graph one complete cycle from (in this case from 0 to 2pi), we know what the rest of the graph looks like.

Can you predict what the graphs of y = 3sin x and y = -2cos x look like? What is the period and amplitude of each of these? SEE NEXT PAGE. . .


The graph of y = 3sin x has the same period as y = sin x but is "stretched" out by a factor of 3. The amplitude is 3 rather than 1.
y = 3sin x
y= sin x
See the next page for the graphs of y = cos x and y = -2cos x.
2
0
-2
-4
x
y
-2π
-1.5π
-1π
-0.5π
0
0.5π
1π
1.5π
2π


The graphs of y = cos x, y = -cos x, and y = -2cos x are shown here. Notice that y = -cos x is a reflection of y = cos x across the x-axis. y = -2cos x is then a vertical stretch of y = -cos x by a factor of 3. The amplitude of y = -2cos x is a = 2 and the period is still 2pi.
y = -2cos x
y = cos x
y = -cos x
What general rules can you make concerning the graphs of y = a sin x and y = a cos x, where "a" is a real number factor? SEE NEXT PAGE
4
2
0
-2
-4
x
y
-2π
-1.5π
-1π
-0.5π
0
0.5π
1π
1.5π
2π

Rules for y = a sin x & y = a cos x.

y = a sin x has the same period and general shape as y = sin x but is stretched out by a factor of "a". In addition, if "a" is negative, the graph is also a reflection across the x-axis (turned upside down). The amplitude will equal the absolute value of "a".

y = a cos x has the same period and general shape as y = sin x but is stretched out by a factor of "a". In addition, if "a" is negative, the graph is also a reflection across the x-axis (turned upside down). The amplitude will equal the absolute value of "a".

What would you predict about the graphs with equations given below?

y = sin 2x

y = cos 4x

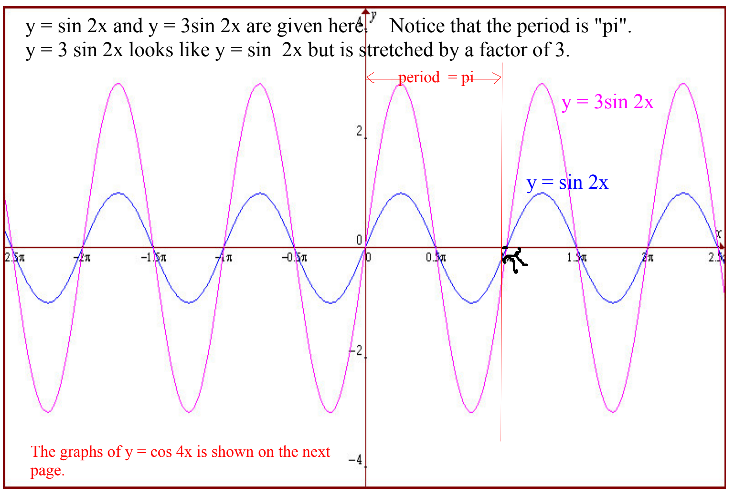
y = 3 sin 2x

You might try constructing a table of values.
For y = sin 2x, pick x-values from 0 to pi at increments of pi/8. For y = cos 4x, pick x-values from 0 to pi/2 at increments of pi/16.

y = 3sin 2x looks much like y = sin 2x except it is . . .?

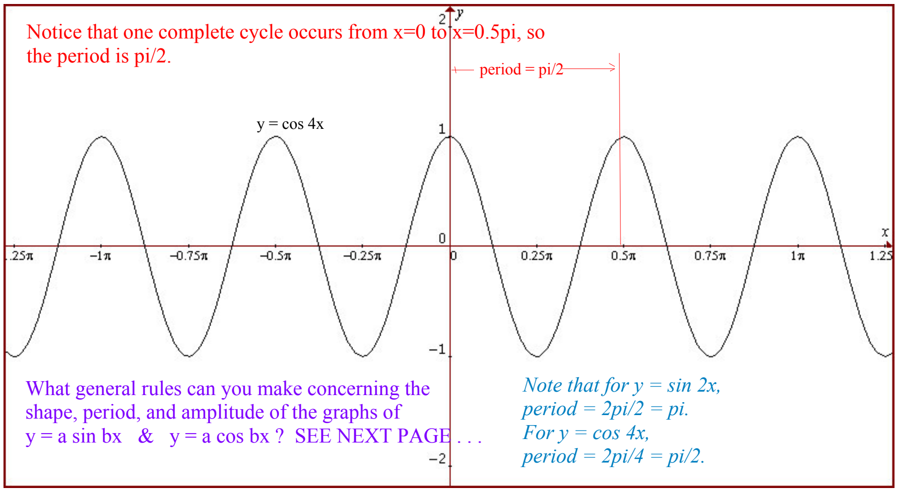
SEE NEXT PAGE. . .

y = sin 2x and y = 3sin 2x are given here. Notice that the period is "pi".
y = 3 sin 2x looks like y = sin 2x but is stretched by a factor of 3.



The graphs of y = cos 4x is shown on the next page.

Notice that one complete cycle occurs from x=0 to x=0.5pi, so the period is pi/2.



What general rules can you make concerning the shape, period, and amplitude of the graphs of y = a sin bx & y = a cos bx ? SEE NEXT PAGE . . .

Note that for y = sin 2x, period = 2pi/2 = pi.
For y = cos 4x, period = 2pi/4 = pi/2.

Rules for y = a sin bx & y = a cos bx.

The graph of y = a sin bx will have a shape like y = sin x but will have a period of 2pi/b and an amplitude equal to the absolute value of "a".

Furthermore, if "a" is negative, the graph is a reflection across the x-axis (ie. it is upside-down).

The graph of y = a cos bx will have a shape like y = cos x but will have a period of 2pi/b and an amplitude equal to the absolute value of "a".

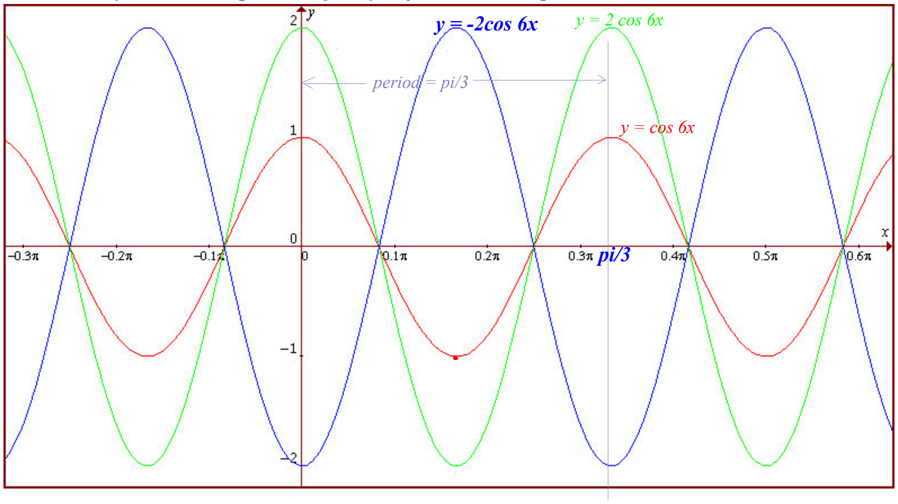
Furthermore, if "a" is negative, the graph is a reflection across the x-axis (ie. it is upside-down).

Example: Predict what the graph of y = -2cos 6x will look like. Then graph it.

$y = -2\cos 6x$ HAS AN AMPLITUDE OF $|-2| = 2$.
IT HAS A PERIOD OF $\frac{2\pi}{6} = \frac{\pi}{3}$.

This will be the upside-down version of y = 2 cos 6x. y = 2cos 6x is the stretched out version of y = cos 6x.
y = cos 6x has a shape like y = cos x but with period = pi/3. SEE NEXT PAGE.

Notice that y = -2cos 6x repeats one full cycle from x=0 to x= pi/3.



Applying More Function Shift Rules to Graphs of Sine and Cosine

In a previous College Algebra or Precalculus course, you should have learned what are called "function shift rules" or perhaps "function translation rules".

Basically, these rules state that given the graph of f(x):

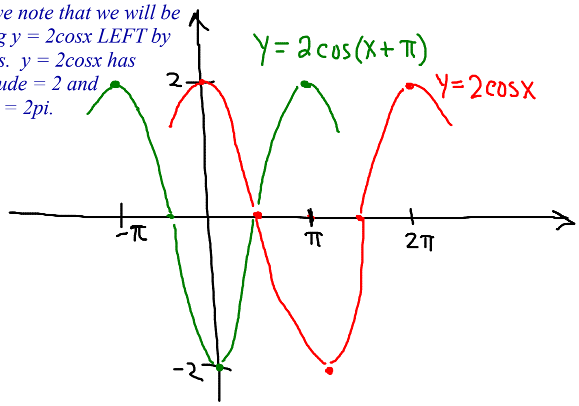
1. The graph of g(x) = f(x + c) will be a shift of "c" units left.
2. The graph of g(x) = f(x - c) will be a shift of "c" units right.
3. The graph of g(x) = f(x) + c will be a shift of "c" units up.
4. The graph of g(x) = f(x) - c will be a shift of "c" units down.
5. The graph of g(x) = -f(x) will be a reflection of f(x) across the x-axis.
6. The graph of g(x) = (c)[f(x)] will be a vertical stretch of f(x) where "c" is a real number multiplying factor.

Note that we have already used rules #5 and #6 in the previous problems.
For a thorough review of these rules from College Algebra, go to http://www.lsc.mnscu.edu/academics/programs/math/sakowski/canotes.htm and click on Section 2.4 .

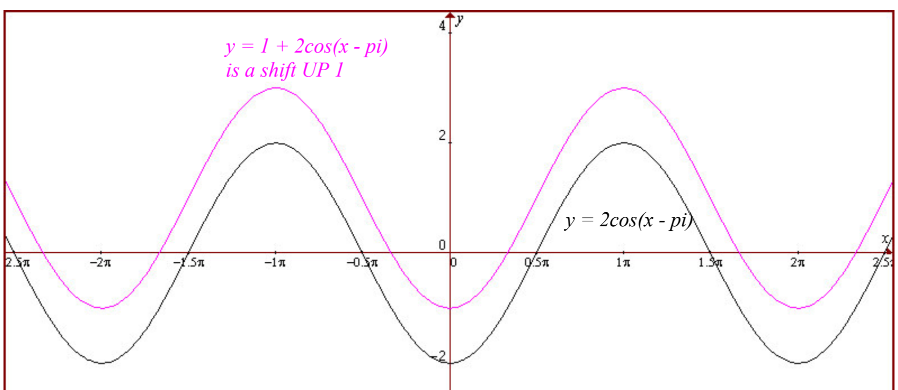
Example: Use the above rules, along with previous rules covered, to graph y = 1 + 2cos(x + pi) . SEE NEXT PAGE . . .

Hint: First identify what shifts occur. Then identify the period and amplitude.

First we note that we will be shifting y = 2cosx LEFT by pi units. y = 2cosx has amplitude = 2 and period = 2pi.



Now, simply shift y = 2cos(x + pi) UP 1 to obtain y = 1 + 2cos(x + pi).
SEE NEXT PAGE . . .

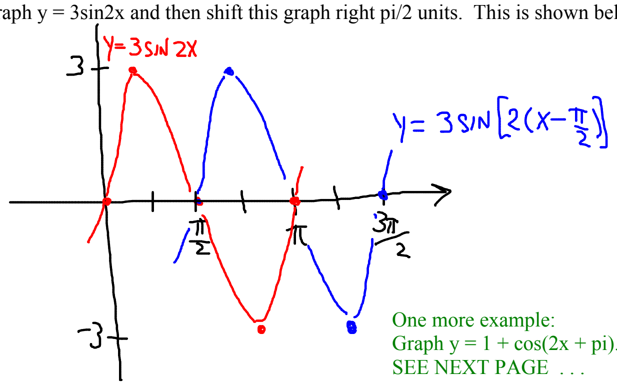


Try to apply function shift rules to graph y = 3sin(2x - pi). Beware!! You must rewrite it in function-shift form 1st! SEE NEXT PAGE. . .

We must rewrite y = 3sin(2x - pi) as y = 3sin[2(x - pi/2)] so that this is now a shift of y = 3sin2x to the RIGHT pi/2 units.

y = 3sin2x has amplitude of 3 and period = 2pi/2 = pi.

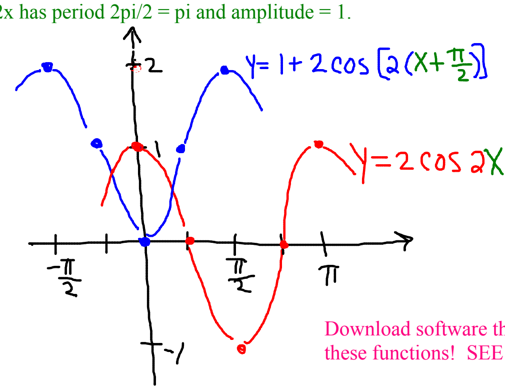
We graph y = 3sin2x and then shift this graph right pi/2 units. This is shown below.



One more example:
Graph y = 1 + cos(2x + pi).
SEE NEXT PAGE . . .

Rewrite y = 1 + cos(2x + pi) as y = 1 + cos[2(x + pi/2)].

This is a shift of y = cos 2x UP 1 and LEFT pi/2. So, graph y = cos 2x and shift it!
y = cos 2x has period 2pi/2 = pi and amplitude = 1.



Download software that will graph these functions! SEE NEXT PAGE

The software program Graphmatica was used to the computer generated graphs posted in these notes. You may download this software at

http://archives.math.utk.edu/software/msdos/calculus/grmat/.html

This is a "zipped" file so you will need to unzip it with Winzip or its equivalent. Winzip is a free download available at http://www.winzip.com/ddchome.htm

A Graphmatica help guide is available at http://www.lsc.mnscu.edu/academics/programs/math/sakowski/caghelp.html

NOTE: You will still need to know how to graph trig functions "by hand"! Use the Graphmatica program to check results of functions that you graph by hand.

Do the exercises for Section 6.4 - Read the text!