Welcome to 6.5 - Graphs of Other Trigonometric Functions

In this section, we will learn how to graph tangent, cotangent, cosecant, and secant functions. We will also graph "damped" sine and cosine functions.

Let's start by graphing y = tan x

First note that y = tan x = (sin x)/(cos x) is not defined whenever cos x = 0, because division by zero occurs. cos x = 0 if x = +/-pi/2, +/-3pi/3, +/-5pi/2, . . .

We also need points on each side of each of these x-values.

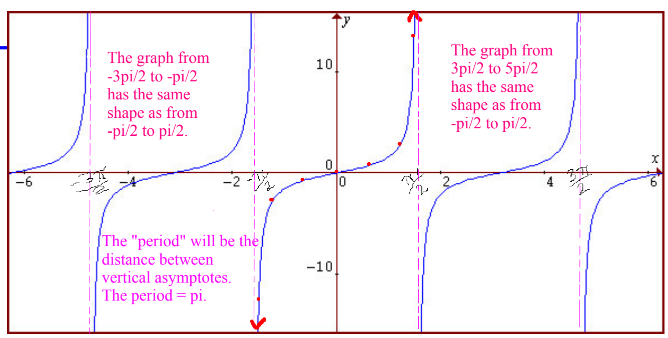
Find the y-values that correspond to each of the x-values given here and then plot the points. Complete the graph by drawing smooth curves through the points.

Note that you should use your calculator to find the y-values and the x-values are given in RADIANS, so use radian mode.

What do you think the graph looks like for x-values from pi/2 to 3pi/2?

x	y
-1.5	
-1.3	
-1.0	
-0.5	
0	
0.5	
1.0	
1.3	
1.5	

x	y
-1.5	-14.1
-1.3	-3.6
-1.0	-1.6
-0.5	-0.5
0	0
0.5	0.5
1.0	1.6
1.3	3.6
1.5	14.1



Note that the curves NEVER cross the vertical lines x=+/- pi/2, +/-3pi/2, . . . These vertical lines make up vertical "asymptotes" since the curves "take off" to infinity and negative infinity for x-values near them.

Also note that between each set of vertical asymptotes, the curve will have a shape identical to the curves between any other set of vertical asymptotes.

This means if you graph one "cycle", you know what the rest of the cycles look like.

Procedure To Graph Tangent and Cotangent Functions

To graph tangent or cotangent functions, all you have to do is

1. Determine what values of the angle result in division by zero. Vertical asymptotes will occur at these values. Plot these vertical asymptotes.

2. Plot a few points between two consecutive vertical asymptotes to establish the shape of one cycle of the graph.

3. The rest of the graph (between all the other consecutive vertical asymptotes) will have shapes identical to the first cycle plotted.

*The period = the distance between two consecutive vertical asymptotes.

Example: Graph y = 2cot 4x

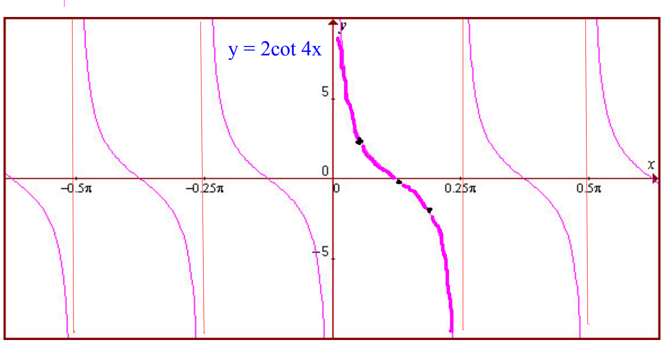
First note that we may write this function as y = 2(cos 4x)/(sin 4x).

The vertical asymptotes occur when sin 4x = 0. This occurs when 4x = 0, 4x = +/-pi, 4x = +/-2pi, . . . which means x = 0, x = +/- pi/4, x = +/- pi/2, . . .

SEE NEXT PAGE. . .

x	y
0.2	1.94
0.6	-2.2
0.4	-.1

Notice that both the tangent and cotangent functions have identical "s" shaped curves separated by vertical asymptotes.

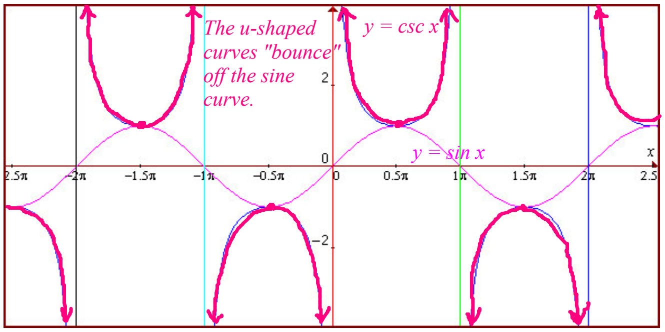


Graphs of Cosecant and Secant

The basic cosecant and secant functions may be graphed fairly easily if you just use the fact that csc x = 1/sin x and sec x = 1/cos x.

Here is a graph of y = csc x. A graph of y = sin x is also shown.

Note that since csc x = 1/(sin x), vertical asymptotes occur whenever sin x = 0 which is at x = 0, +/- pi, +/- 2pi, etc. The csc x graph "bounces" off the sin x curves.



Procedure to graph Cosecant and Secant functions

1. Graph the related Sine or Cosine function.

2. Identify where the vertical asymptotes occur. This is where division by zero occurs.

3. Sketch in the U-shaped cosecant or secant curves that "bounce" off the related sine or cosine curve.

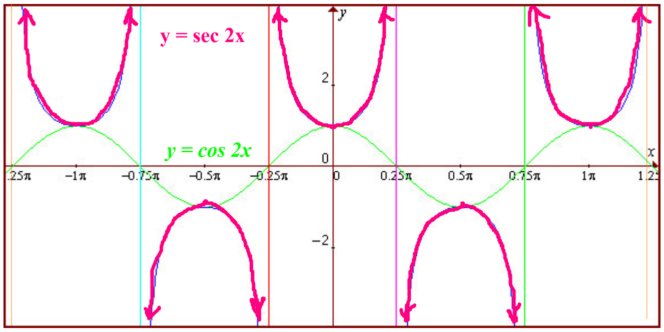
Example: Graph y = sec 2x (Note: sec 2x = 1/cos 2x)

A. Graph y = cos 2x (See Next Page)

B. Vertical asymptotes occur when cos 2x = 0. This occurs when 2x = +/-pi/2, 2x = +/- 3pi/2, 2x = +/- 5pi/2, etc. which means x = +/-pi/4, x = +/-3pi/4, x = +/- 5pi/4.

C. The U-shaped secant curves "bounce off" the sine curve in between the vertical asymptotes.

The graph of y = cos 2x is shown below in green and the vertical asymptotes are also shown. The U-shaped curves that make up the graph of y = sec 2x are shown "bouncing off" the sine curve in between each pair of vertical asymptotes.



Why do the U-shaped curves of y = sec 2x "bounce off" the cosine curve the way they do? See Next Page for answer.

Why do the U-shaped curves of y = sec 2x "bounce off" the cosine curve the way they do?

Answer: At the values of x that make y = cos 2x equal to 1, the values of y = sec 2x will equal 1/1 = 1. At the values of x that are close to the vertical asymptotes, the values of y = cos 2x get closer to zero. This means that the values of y = sec 2x will get larger and larger since sec 2x = 1/cos 2x. When we divide 1 by a very small number, the result is a very large number. In fact, the curves of y = sec 2x take on infinitely large positive and negative values if x is a value very, very close to one of the vertical asymptotes.

Graphs of Damped Trigonometric Functions

To get an idea of what a simple "damped" trigonometric function is, plot the function y = x sin x and use the x-values

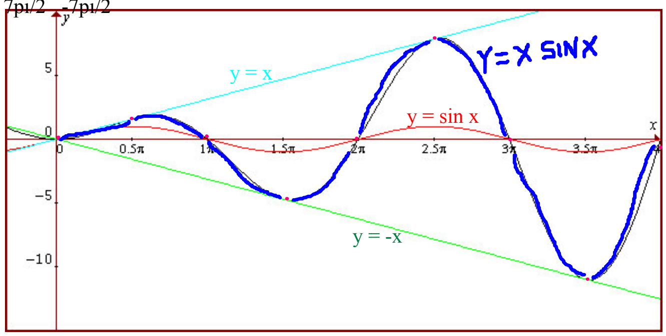
x = 0, pi/2, pi, 3pi/2, 2pi, 5pi/2, 3pi, 7pi/2, 4pi

x	y = x sinx
0	0
pi/2	pi/2(1)
pi	0
3pi/2	-3pi/2
2pi	0
5pi/2	5pi/2
3pi	0
7pi/2	-7pi/2

The graph of y = x sin x is shown below. Also, the graph of y = sin x is shown, as well as the graphs of y = x & y = -x.

Notice that the graph of y =x sin x is "bounded" by the graphs of y = x and y = -x.

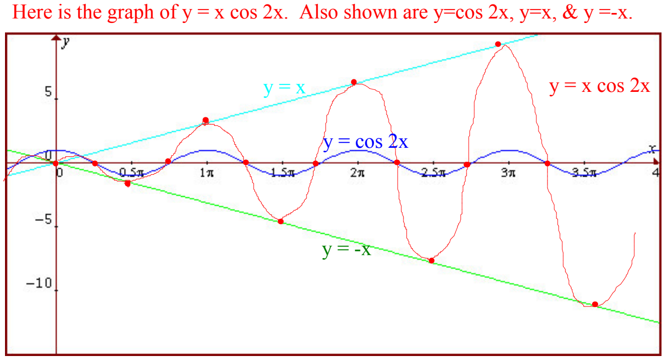
Can you think of a general approach to graphing y = x cos bx, or y = x sin bx where "b" is a coefficient on x? SEE NEXT PAGE



Procedure to Graph y = x sin bx or y = x cos bx.

1. Plot y = sin bx or y = cos bx.
2. Plot y = x and y = -x.
3. "Stretch" the curves of y = sin bx or y = cos bx out at their maximum or minimum values to meet the curve of y = x or y = -x.
4. If ANY of y = x, y = -x, y = sin bx, or y = cos bx have zero y-values, the graph of y = xsin bx or y = xcos bx will have an x-intercept.

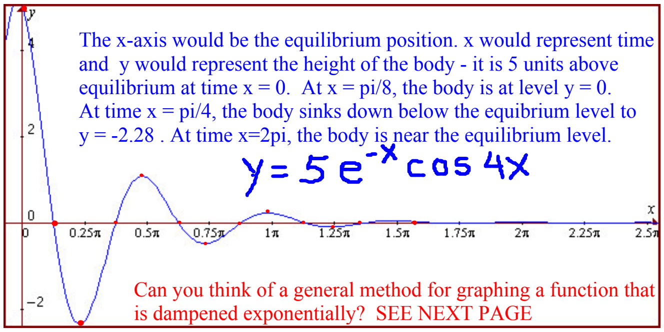
Here is the graph of y = x cos 2x. Also shown are y=cos 2x, y=x, & y =-x.



Functions With Exponential Damping

If we plot the motion of a body that bounces upon a spring, we see that the body goes up and down, but due to friction, the body eventually bounces less and less and eventually comes to rest at a point of equilibrim.

An example of such motion is shown in the graph of y = 5e^(-x) cos 4x below.



How to Graph Exponentially Damped Functions of the Forms Given Here:

$y = e^{ax} \sin bx$ $\qquad$ $y = e^{ax} \cos bx$

1. Graph $y = e^{ax}$.
2. Graph $y = -e^{ax}$.

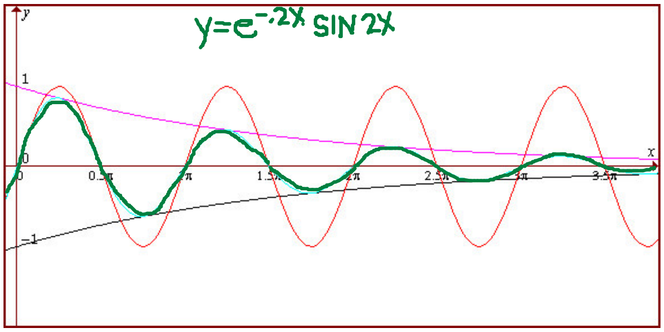
These exponential functions form the "boundaries" for the exponentially damped functions, in the same way that y=x and y=-x were boundaries in the previous problems.

3. Graph the sine or cosine function y = sin bx or y = cos bx.

4. "Stretch" the maximum and minimum values of the sine or cosine functions out to the boundaries formed by the exponential function graphs.

Example: How would you graph $y = e^{-0.2x} \sin 2x$?

The graphs of y=e^(-.2x), y = -e^(-.2x), and y = sin 2x are plotted. The the sine curve is "sandwiched" between the two exponential curves.



Why should I know how to graph these "by hand" when a software program like Graphmatica will graph them so easily? SEE NEXT PAGE FOR ANSWER>>>

You should know "how" to graph it by hand so that you may simply look at an equation and be able to predict what the graph behaves like.

For example, you should be able to look at the equation of y= 10 + 0.1cos x and know that its graph is a shift of y = 0.1cosx UP 10 and has an amplitude of only 0.1 .

The graph generated on Graphmatica is shown here: