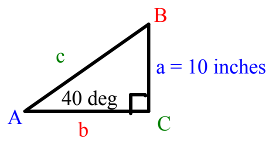
Welcome to Section 6.7 - Applications

In this section, you will apply principles learned earlier to solve application problems that involve trigonometric functions.

Solving Right Triangles

To "solve" a right triangle, means to find all the values of all the side lengths and all the angles of the triangle by using the side lengths and angles that are given.

Example: "Solve" the right triangle shown here where A = 40 degrees & a = 10 inches.

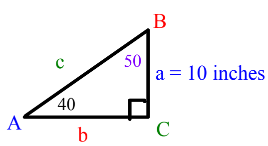


Note that the angles are labeled A, B, C, and the sides opposite these respective angles are labeled a,b, and c.

Solution:

We can find angle B by noting that the sum of all angles of a triangle must be 180 degrees. By subtraction of 180 - (90 + 40) we get B = 50 deg.

How can we find sides "b" or "c"? SEE NEXT PAGE . . .



We know that TAN 40 = (Opposite)/(Adjacent) = 10/b.

This results in TAN 40 = 0.8391 = 10/b .

Multiplying both sides by "b" results in 0.8391 b = 10.

Now, divide both sides by 0.8391 to get b =10/0.8391 = 11.92 inches

You may find the last side length by either using the Pythagorean Theorem OR by using the relationship SIN 40 = 10/c. The answers we get are shown below:

$$c = \sqrt{a^2+b^2} = \sqrt{10^2 + 11.92^2} \approx 15.56 \text{ inches}$$

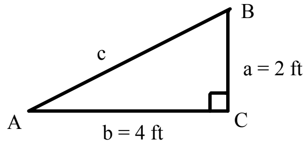
$$\sin 40^\circ = \frac{10}{c}$$

$$.6428 = \frac{10}{c}$$

$$.6428\,c = 10 \rightarrow c = \frac{10}{.6428} \approx 15.56 \text{ inches}$$

Note that both of these methods involved rounding. In order to get a more exact answer, solve for the side in terms of the trig function. and THEN round after you get the final answer. For example, c = 10/(SIN 40) = 15.5572382686 = 15.56 (rounded)

Example: Solve the right triangle shown below.



Solution: In this case we may solve for the hypotenuse with the Pythagorean Theorem.

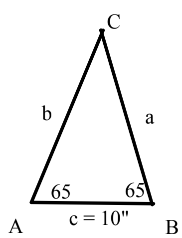
$$c = \sqrt{2^2 + 4^2} = \sqrt{20} \approx 4.47 \text{ ft.}$$

Now, find the angles by using the inverse trig functions.

TAN A = 2/4   Thus, Arctan (2/4) = A = 26.57 degrees (rounded)

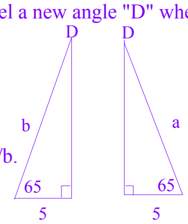
We can find the final angle B by subtracting 180 - (90 + 26.57) = 63.43
so B = 63.43 degrees.

Example: The triangle given below is an isosceles triangle because the two angles at the base are equal to 65 degrees. Solve this triangle.



Solution: Divide this triangle into two equal right triangles as shown below. Then solve for "a" or "b". Since this is an isosceles triangle, "a" = "b".

We also label a new angle "D" where D = (1/2)C



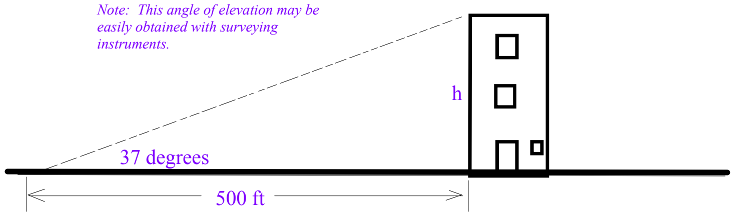
We can find "b" by noting that COS 65 = 5/b.

b(COS 65) = 5,   b = 5/(COS 65) = 11.83

"a" also = 11.83 We may find angle D by subtraction of 180 - (90 + 65) = 25 degrees.

Angle C will be twice D so angle C = 50 degrees.

Example: Find the height of a building given that the measurements below are taken.
The angle of 37 degrees is known as the "angle of elevation".

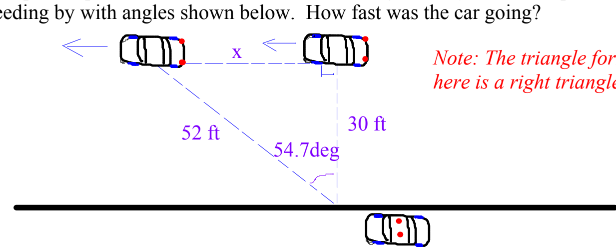


Solution: If we let "h" represent the height of the building, TAN 37 = h/500, so 500(TAN 37) = h = 376.777 ft . How could you convert this to feet and inches?

0.777 ft = 0.777(12 inches) = 9.3 inches, which we round to 9 inches.

Thus, h = 376 feet, 9 inches.

Example: A police officer takes two radar readings exactly 1 second apart of a car speeding by with angles shown below. How fast was the car going?



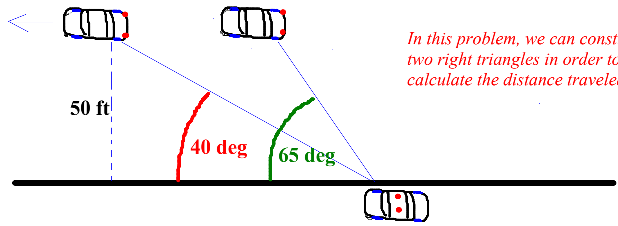
Note: The triangle formed here is a right triangle.

Solution: If we let "x" represent the distance traveled in 1 second, we may find x either with trigonometry or the Pythagorean Theorem.

By the Pythagorean Theorem, we get x = $\sqrt{52^2-30^2}$ = 42.47 ft .

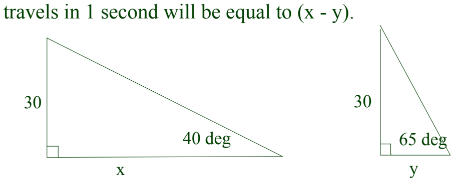
This car is going 42.47 ft per second. If we multiply by (1mi)/(5280 ft) and then multiply by (3600 seconds)/(1 hour), we get a speed of about 29 miles per hour.

Example: A police officer takes two radar readings exactly 1 second apart of a car speeding by with angles shown below. How fast was the car going?



In this problem, we can construct two right triangles in order to calculate the distance traveled.

Solution: We have two right triangles, as labeled below. The distance that the car travels in 1 second will be equal to (x - y).



Tan 40 = 50/x
x = 50/(tan 40) = 59.59

Tan 65 = 50/y
y = 50/(tan 65) = 23.32

x - y = 36.27 feet

Now convert to M.P.H. - SEE NEXT PAGE . . .

Here we convert feet per second into miles per hour.

$$\frac{36.27\ \text{ft}}{1\ \text{sec}} \times \frac{1\ \text{MILE}}{5280\ \text{ft}} \times \frac{3600\ \text{sec}}{1\ \text{HOUR}} = 24.72\ \text{MPH}$$

Notice that the feet units cancel and the second units cancel, leaving only the units of miles in the numerator and hours in the denominator.

We can round this to 25 M.P.H.

Applications Involving Bearings

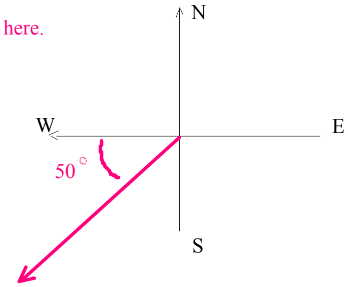
"Bearings", as used in trigonometric applications, refer to the acute angle a line of sight makes with another fixed North-South or East-West line.

The format is easiest to show by example.

Examples of Bearings

W 50 S means that you face West, and then veer off 50 degrees to the South.
See next page for illustration.

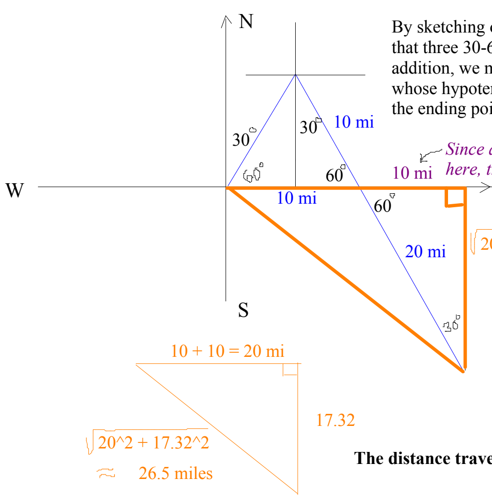
W 50 S is shown here.



Example

A pilot flies 10 miles with bearings N 30° E and then turns and flies 30 miles with bearings S 30° E. How far from the original starting point is the pilot?

In this problem, a sketch is absolutely necessary! SEE NEXT PAGE



By sketching out the path (in blue), we see that three 30-60-90 triangles are formed. In addition, we may construct a right triangle whose hypotenuse will equal the distance of the ending point from the starting point.

Since a 30-60-90 triangle is formed here, this side length is 1/2 of 20.

√(20^2 - 10^2) = 17.32 (rounded)
OR sqrt(300)

Note that the 20 mile side length is found by subtracting 10 from 30. Because an equilateral triangle is formed, we know that its 3 side lengths are all equal to 10 mi.

The distance traveled is about 26.5 miles.

That's all for this Chapter. Do the homework!