Section 7.1 - Using Fundamental Identities

Trigonometric identities are used to accomplish the following:

1. Rewrite a trigonometric expression in a different form.

2. Rewrite a trigonometric equation in a different form so that we may algebraically solve the equation.

Example: Rewrite the product (sin u)(cot u) in its simplest form.

$(\sin u)(\cot u)$

$= (\cancel{\sin u})\left(\dfrac{\cos u}{\cancel{\sin u}}\right)$

$= \cos u$

Note: We replace cot u with its equivalent cot u = (cos u)/(sin u)

Example: Apply the cofunction identity sin(pi/2 - u) = cos u to evaluate sin (pi/8) given that cos (3pi/8) = 0.38268 (rounded).

Since pi/2 - 3pi/8 = pi/8, sin(pi/2 - u) = sin pi/8 where u = 3pi/8.

By the identity, sin(pi/8) = sin(pi/2 - u) = cos u = cos 3pi/8 = 0.38268

Example: Combine the two fractions below and simplify.

$\dfrac{\sin u}{\cos u} + \dfrac{\cos u}{\sin u}$

$= \dfrac{\sin u}{\cos u} \cdot \dfrac{\sin u}{\sin u} + \dfrac{\cos u}{\sin u} \cdot \dfrac{\cos u}{\cos u}$

First, we must write these fractions with the common denominator of (sin u)(cos u).

$= \dfrac{\sin^2 u}{\sin u \cos u} + \dfrac{\cos^2 u}{\sin u \cos u}$

$= \dfrac{\sin^2 u + \cos^2 u}{\sin u \cos u}$

We can combine the fractions together.

$= \dfrac{1}{\sin u \cos u}$

Since $\sin^2 u + \cos^2 u = 1$, the numerator = 1.

$= \dfrac{1}{\sin u} \cdot \dfrac{1}{\cos u} = \csc u \cdot \sec u$

Because 1/sin u = csc u & 1/cos u = sec u

$\dfrac{\sin u}{\cos u} + \dfrac{\cos u}{\sin u} = \csc u \cdot \sec u$

What does this imply about the sum tan u + cot u?

Since sin u/cos u = tan u and cos u/sin u = cot u, what we have shown is

tan u + cot u = (csc u)(sec u)

Using Identities to Simplify Expressions

The above statement, tan u + cot u = (csc u)(sec u) is called an "identity" because the left side of the statement may be shown to equal the right side of the statement by applying other identities.

We will simplify complex expressions by

A) Making substitutions using existing identities.

B) Use algebra to combine fractions, factor expressions, multiply expressions, etc.. in order to make an expression simpler.
In the last example, we combined fractions, made substitutions by applying existing identities, and then simplified some more.

*Note: In order to simplify expressions, you will need a complete list of known identities to work with. A list of some common identities is given on the next page.

Basic Identities

You may want to print these out!

$$\tan u = \frac{\sin u}{\cos u} \qquad \cot u = \frac{\cos u}{\sin u} \qquad csc\ u = \frac{1}{\sin u}$$

$$\sec u = \frac{1}{\cos u} \qquad \cot u = \frac{1}{\tan u} \qquad \tan u = \frac{1}{\cot u}$$

Pythagorean Identities

$\sin^2 u + \cos^2 u = 1$ $\quad\cos^2 u = 1 - \sin^2 u$ $\quad\sin^2 u = 1 - \cos^2 u$

$\tan^2 u = \sec^2 u - 1$ $\quad\sec^2 u = \tan^2 u + 1$

$\cot^2 u = \csc^2 u - 1$ $\quad\csc^2 u = \cot^2 u + 1$

Cofunction Identities

$$\cos\left(\frac{\pi}{2} - u\right) = \sin u \qquad \sin\left(\frac{\pi}{2} - u\right) = \cos u \qquad \cot\left(\frac{\pi}{2} - u\right) = \tan u$$

$$\tan\left(\frac{\pi}{2} - u\right) = \cot u \qquad csc\left(\frac{\pi}{2} - u\right) = \sec u \qquad \sec\left(\frac{\pi}{2} - u\right) = csc\ u$$

Negative Angle Identities

sin(-u) = - sin u $\quad$ cos(-u) = cos u $\quad$ tan(-u) = - tan u

sec(-u) = sec u $\quad$ csc(-u) = -csc u $\quad$ cot(-u) = - cot u

Example: Simplify $\csc^2 u - (\csc^2 u)(\cos^2 u)$
There are two different ways to approach this.

METHOD 1 - Factor out csc^2 first.

$$= \csc^2 u\,(1 - \cos^2 u)$$
$$= \csc^2 u\,(\sin^2 u)$$
$$= \frac{1}{\sin^2 u}(\sin^2 u)$$
$$= 1$$

Note that both methods result in the same simplified result. The first method is a bit quicker.

METHOD 2 - Substitute (1 - sin^2 u) in for cos^2 u first.

$$\csc^2 u - (\csc^2 u)(\cos^2 u) = \csc^2 u - (\csc^2 u)(1 - \sin^2 u)$$
$$= \csc^2 u - \left[\csc^2 u - (\csc^2 u)(\sin^2 u)\right]$$

Multiply out the csc^2 term by the (1 - sin^2) term

$$= \cancel{\csc^2 u} - \cancel{\csc^2 u} + \csc^2 u \sin^2 u$$

Remove the parentheses. Get the correct signs!

$$= \csc^2 u \sin^2 u \quad = 1$$

The csc^2 and sin^2 terms cancel each other.

Example: Simplify $\frac{1}{\cos u} - \cos u$ and show that it equals (tan u)(sin u).

$$\frac{1}{\cos u} - \frac{\cos u}{1}$$
$$= \frac{1}{\cos u} - \frac{\cos u}{1} \cdot \frac{\cos u}{\cos u}$$
$$= \frac{1}{\cos u} - \frac{\cos^2 u}{\cos u}$$
$$= \frac{1 - \cos^2 u}{\cos u}$$

Combine these terms by writing them both as fractions, getting common denominators, and combining with subtraction.

$$= \frac{\sin^2 u}{\cos u}$$

Apply the Pythagorean Identity here to make a substitution.

$$= \frac{\sin u}{\cos u} \cdot \sin u$$

Split off a sine factor.

$$= (\tan u)(\sin u)$$

Apply the basic identity for tangent.

Example: Simplify csc u - (cot u)(cos u) and show that it equals sin u.

We can again begin by writing each of these as fractions by applying identities for csc u and cot u.

$$\csc u - (\cot u)(\cos u)$$

$$= \frac{1}{\sin u} - \left(\frac{\cos u}{\sin u}\right)(\cos u)$$

$$= \frac{1}{\sin u} - \frac{\cos^2 u}{\sin u}$$

Whatdayaknow? These have the same denominators! You may combine them.

$$= \frac{1 - \cos^2 u}{\sin u}$$

Now, apply one of the Pythagorean Identities to the numerator, and then cancel out sine terms.

$$= \frac{\sin^2 u}{\sin u} = \sin u$$

Can you think of a way we can use our calculator to "check" our work?
SEE NEXT PAGE. . .

We can check that csc u - (cot u)(cos u) = sin u, by letting "u" equal some arbitrary non-zero value and then plug this value into both sides of this equation.

We could let u=4. It doesn't even matter if we use degrees or radians.

Using u = 4 radians, sin 4 = -.756802 and

csc 4 - (cot 4)(cos 4) = 1/sin 4 - (cos 4)/(tan 4) = -.756802

Note that because our calculator does not have keys for cotangent or cosecant, we must calculate cot 4 as 1/(tan 4) and calculate csc 4 as 1/sin 4.

Using a calculator to check the result this way isn't 100% conclusive since it is possible that just by coincidence, u = 4 is a value that happens to work. This is highly unlikely, but possible. For example, if we let u = pi to check to see if sin u = tan u, we would get sin pi=0 and tan pi = 0. However, if we pick u = 3 radians, we get sin 3 = .1411 and tan 3 = - .1425 . Thus, by avoiding using values of u like 0,pi,pi/2,pi/4 and instead picking "odd" values of u like 2,3,25, etc, this method is much less likely to erroneously show your result true by "coincidence".

Do the homework problems for this Section!!!!