Welcome to Section 7.2 - Verifying Trigonometric Identities

A trigonometric "identity" is a statement that equates one trigonometric expression with another.

For example, (cos u)(tan u) = sin u is an identity because we may show that

(cos u)(tan u)

= (cos u)[(sin u)/(cos u)]

= sin u

How to Verify a Trigonometric Identity

To verify a trigonometric identity, you should

1. Choose the more complex side of the equation, and then simplify it and get it into the form of the other side of the equation by
   A) Algebraically combining terms & fractions, factoring, multiplying, etc,
   B) Replacing trigonometric terms with their equivalent by applying a existing trigonometric identity.
2. If you are unable to easily simplify this side of the equation simplify the other side. Sometimes one side is easy to work on, and the other is very difficult.

Example: Verify the identity $\tan x - \sin^2 x \tan x = \sin x \cos x$.

First, we must choose a side to simplify. We choose the left side since it is more complex than the right side. We then simplify the left side and (hopefully) obtain the right side.

$\tan x - \sin^2 x \tan x$

$= \tan x\,(1 - \sin^2 x)$ We can apply a Pythagorean Identity.

$= \tan x\,(\cos^2 x)$

$= \dfrac{\sin x}{\cos x} \cdot \cos^2 x$ Replace tan x with (sin x)/(cos x).

$= (\sin x)(\cos x)$ Now, cancel out one of the cosine factors.

What if it is impossible to prove an identity and you can actually show that the left side is NOT equal to the right side by substituting an angle value in with your calculator? ANSWER: It's NOT an identity - there is a misprint! And yes, this has happened on rare occasion!

Example: Verify the identity csc u = (cot u)(cos u) + sin u.

It is fairly obvious here that the right side is more complex. Thus, we must simplify the right side and (hopefully) obtain csc u.

$(\cot u)(\cos u) + \sin u$ *Replace cot u with its equivalent.*

$= \left(\frac{\cos u}{\sin u}\right)(\cos u) + \sin u$

$= \frac{\cos^2 u}{\sin u} + \frac{\sin u}{1}$ *Write both terms as fractions.*

$= \frac{\cos^2 u}{\sin u} + \frac{\sin u}{1} \cdot \frac{\sin u}{\sin u}$ *Obtain common denominators.*

$= \frac{\cos^2 u + \sin^2 u}{\sin u}$ *Combine the fractions.*

$= \frac{1}{\sin u}$ *Apply a Pythagorean Identity to the numerator.*

$= \csc u$

Example: Verify that $1 + \sin u = \frac{\cos^2 u}{1 - \sin u}$.

Here, we are not quite sure which side to choose. If we take a close look at the right side, we see that the cosine squared term may be replaced with (1 - sin^2 u). 1 - sin^2 u in turn, may be factored as (1 + sin u)(1 - sin u). We cancel and were done.

$\frac{\cos^2 u}{1 - \sin u}$

$= \frac{1 - \sin^2 u}{1 - \sin u}$

$= \frac{(1+\sin u)\cancel{(1-\sin u)}}{\cancel{1-\sin u}}$

$= 1 + \sin u$

In this problem, you could also show that the left side may be transformed into the right side.

If we start with 1 + sin u, we write this as a fraction (1+ sin u)/1. We obtain a denominator of 1 - sin u, by multiplying the numerator AND denominator by (1 - sin u). From there, we may obtain the right side.

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$1+\sin u$

$= \dfrac{1+\sin u}{1}$ Write this as a fraction with a denominator of 1.

$= \dfrac{(1+\sin u)}{1} \cdot \dfrac{(1-\sin u)}{(1-\sin u)}$ Multiply to obtain the denominator of (1 - sin u).

$= \dfrac{1-\sin^2 u}{1-\sin u}$ Multiply out the terms in the numerator.

$= \dfrac{\cos^2 u}{1-\sin u}$ Apply a Pythagorean identity to the numerator.

Example: Verify that $1+2\tan^2 u = \sec^4 u - \tan^4 u$.

Here, we choose to work on the right side since it is more complex. We start by using the fact that $a^4 - b^4 = (a^2+b^2)(a^2-b^2)$.

$\sec^4 u - \tan^4 u$ We factor this as a difference of squares.

$= (\sec^2 u + \tan^2 u)(\sec^2 u - \tan^2 u)$ We apply a Pythagorean identity.

$= (\tan^2 u + 1 + \tan^2 u)(\tan^2 u + 1 - \tan^2 u)$ Simplify.

$= (2\tan^2 u + 1)(1)$ Simplify.

$= 2\tan^2 u + 1$

That's all for this section. Do the homework problems!