Welcome to Section 7.4 - Sum & Difference Formulas

The sum and difference formulas for sine, cosine, and tangent are given here:

sin(u + v) = sin u cos v + cos u sin v
sin(u - v) = sin u cos v - cos u sin v

cos(u + v) = cos u cos v - sin u cos v
cos(u - v) = cos u cos v + sin u cos v

$$\tan(u + v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

$$\tan(u - v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

These formulas may be derived by using the distance formula applied to x = cos u, y = sin u for the terminal point of the angle formed by angle u on the unit circle.

We may use these formulas to rewrite trigonometric expressions.

Example: Use an angle sum identity to find the "exact" value of sin(7pi/12).

You may rewrite sin(7pi/12) as sin(u + v) where "u" and "v" are special-case angles corresponding to 30 deg, 45 deg, 60 deg, etc.

What should you pick for "u" and "v"? SEE NEXT PAGE . . .

Example: Use an angle sum identity to find the "exact" value of sin(7pi/12).

$$\sin\left(\frac{7\pi}{12}\right) = \sin\left(\frac{3\pi}{12} + \frac{4\pi}{12}\right)$$

We choose 3pi/12 and 4pi/12 because they simplify to pi/4 and pi/3 which are "special-case" angles, of which we know the exact values.

$$= \sin\frac{3\pi}{12}\cos\frac{4\pi}{12} + \cos\frac{3\pi}{12}\sin\frac{4\pi}{12}$$

$$= \sin\frac{\pi}{4}\cos\frac{\pi}{3} + \cos\frac{\pi}{4}\sin\frac{\pi}{3}$$

$$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$$

$$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$$

$$= \frac{\sqrt{6}+\sqrt{2}}{4}$$

Note: The "exact" value is the an answer that does not contain a rounded off decimal. In this case, we must write the answer in radical form.

We can "check" our answer simply by finding the decimal equivalent of sin(7pi/12) and of the answer. Both have a decimal equivalent of 0.9659258 . . .

Example: Find the exact value of tan(255°) by using either an angle difference or angle sum identity.

Here, we must rewrite 255 degrees as the sum or difference of two "special case" angles. In this case, we have two obvious choices.

255 = 210 + 45 OR
255 = 300 - 45

Thus, tan(255) = tan(210 +45) OR tan (255) = tan(300 - 45)
If we evaluate both of these by using the sum & difference formulas, we get

tan(255) = tan(210 + 45)

$$= \frac{\tan 210^\circ + \tan 45^\circ}{1 - (\tan 210^\circ)(\tan 45^\circ)}$$

You must construct triangles corresponding to 210 deg and 45 deg, find sine and cosine, and use the identity tan u = (sin u)/(cos u) in order to find the values of these tangent functions.

$$= \frac{\frac{1}{\sqrt{3}} + 1}{1 - \left(\frac{1}{\sqrt{3}}\right)(1)}$$

Now, multiply top and bottom by the square root of 3 to get

$$\frac{1+\sqrt{3}}{\sqrt{3}-1}$$

Example: Verify the identity $\sin u \sin v = \frac{1}{2}[\cos(u-v) - \cos(u+v)]$.

We start with the more complex side and apply angle sum and difference formulas to get

$$\frac{1}{2}[\cos(u-v) - \cos(u+v)]$$

$$= \frac{1}{2}[\cancel{\cos u \cos v} + \sin u \sin v - (\cancel{\cos u \cos v} - \sin u \sin v)]$$

$$= \frac{1}{2}[2 \sin u \sin v]$$

Note that the cos u cos v terms cancel and the second sin u sin v term is added to the first.

$$= \sin u \sin v$$

Example: Use an angle sum or difference identity to solve the equation sin(x + pi) = 1/2.

By the angle sum identity, sin(x + pi) = sin x cos(pi) + cos x sin(pi). We let this equal 1/2 and replace sin(pi) with 0 and cos(pi) with -1 to get

1/2 = (sin x)(-1) + (-1)(0) OR
1/2 = -sin x OR
-1/2 = sin x

After getting to the point -1/2 = sin x, you may wish to construct reference triangles with corresponding sine values of -1/2.

The solutions for this are x = 7pi/6 + 2n(pi) and x = 11pi/6 + 2n(pi).

Example: Verify that $\sin(2x) = 2\sin x \cos x$.

By the angle sum identity

$$\begin{aligned} &\sin(2x) \\ &= \sin(x + x) \\ &= \sin x \cos x + \sin x \cos x \\ &= 2 \sin x \cos x \end{aligned}$$

Note: In this problem "u" = "x" AND "v" = "x".

That's all for this section. Do The Homework Problems!