Section 8.2 - Law of Cosines

It may be shown, by using right triangle trigonometry and algebra, that for ANY triangle with angles A, B, & C and corresponding sides a, b, & c

$$a^2 = b^2 + c^2 - 2bc(\cos A)$$

$$b^2 = a^2 + c^2 - 2ac(\cos B)$$

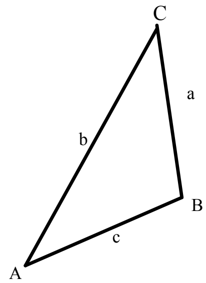
$$c^2 = a^2 + b^2 - 2ab(\cos C)$$

Alternatively, we may solve the above for a,b, and c to get

$$a = \sqrt{b^2 + c^2 - 2bc(\cos A)}$$

$$b = \sqrt{a^2 + c^2 - 2ac(\cos B)}$$

$$c = \sqrt{a^2 + b^2 - 2ab(\cos C)}$$



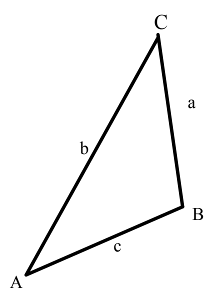
We may also, solve for any of the angles A, B, and C.
SEE NEXT PAGE . . .

Solving for the angles A, B, & C results in the formulas below. Note that you must use your inverse cosine function on your calculator.

$$A = \arccos\left[\frac{b^2 + c^2 - a^2}{2bc}\right]$$

$$B = \arccos\left[\frac{a^2 + c^2 - b^2}{2ac}\right]$$

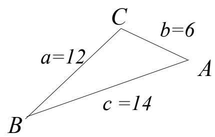
$$C = \arccos\left[\frac{a^2 + b^2 - c^2}{2ab}\right]$$



Note: With the Law of Cosines, we may solve a triangle if we are given only 3 side lengths (SSS) or if we are given only two sides and the angle between the two sides (SAS). This was not the case with the Law of Sines.

Example: A triangle has sides a = 12", b = 6", and c = 14.
Solve the triangle.

Start with a sketch of a triangle with lengths as given and corresponding labels for angles.



From the Law of Sines, we obtain angle A with the formula

$$A = \text{ARCCOS}\left[\frac{b^2+c^2-a^2}{2bc}\right]$$

$$A = \text{ARCCOS}\left[\frac{6^2+14^2-12^2}{2(6)(14)}\right] = 58.41 \text{ degrees}$$

WHAT ? You didn't get this answer??? SEE NEXT PAGE FOR TIPS ON EVALUATING

$$A = \text{ARCCOS}\left[\frac{6^2+14^2-12^2}{2(6)(14)}\right] = 58.41 \text{ degrees}$$

Start by evaluating 6^2 + 14^2 - 12^2 = 88

Next, enter the following ÷ (2 × 6 × 14) =
and you should now have a result of 0.523809524 .

If you have an older type scientific calculator, simply hit the inverse cosine function key $\cos^{-1}$
to get 58.41 degrees.

If you have a newer scientific calculator, you will have to store the result 0.523809524 either in your calculators memory or write it down, and then press the inverse cosine function key and enter in 0.523809524 . We could also enter this in one shot as arccos ((6^2 +14^2 -12^2)/(2 X 6 X 14)) = 58.41 degrees. YOU NEED ALL THOSE PARENTHESES!

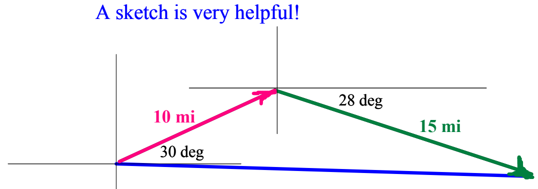
Using the Law of Cosines to find angle C results in
C = arccos[(12^2+6^2-14^2)/(2*12*14)] = 96.38 degrees.

We may find angle B by subtracting 180 - (58.41+96.38) = 25.21 degrees.

Notice that the angles obtained match what we would expect from the sketch!

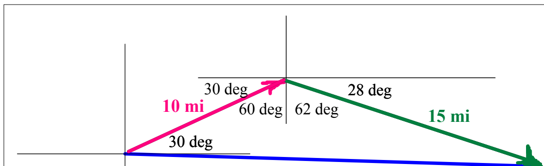
Example: A jet flies 10 miles with a bearing of E 30° N. It then switches direction and flies 15 miles with a bearing of E 28° S. How far away from the point of origin is the jet at this point?

A sketch is very helpful!

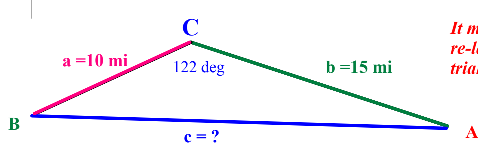


We can use the bearings given to determine the angle that is between the two given side lengths. Once we have two sides and an included angle (SAS), we may use the Law of Cosines to find a third side length. The third side length will be the distance the jet flies.

Start by finding the top angle. SEE NEXT PAGE . . .



The 30 deg angle at the top is an alternate interior angle. By subtraction from 90 we obtain the 60 deg angle. Also, by subtracting 90 - 28, we get the 62 deg angle. The top angle is 60 + 62 = 122 degrees.



It may be helpful to re-lable as an ABC triangle!

By the Law of Cosines from the very first page

$$c = \sqrt{a^2+b^2-2ab\cos C} = \sqrt{10^2+15^2-2(10)(15)\cos(122)}$$

c = 22.00 miles (rounded)

Heron's Area Formula

If we apply the Law of Cosines to a triangle with sides of length a, b, and c, we may show that the area of the triangle is given by

$$\text{AREA} = \sqrt{d(d-a)(d-b)(d-c)}$$

$$\text{WHERE } d = \frac{a+b+c}{2}$$

This seems like a complicated formula, but it is extremely easy to use.

Example: Find the area of a triangle with sides of length a=8", b=5", c=9".

First calculate d. d = (8 + 5 + 9)/2 = 11
Now, simply plug in the quantities a, b, c, & d into the formula.

$$\begin{aligned} \text{AREA} &= \sqrt{11(11-8)(11-5)(11-9)} \\ &= \sqrt{11(3)(6)(2)} \\ &= \sqrt{396} \approx 19.90 \text{ in}^2 \end{aligned}$$

Note: The units are "square inches" since this is an area.

That's all for this section - Do the Homework!!!!