Project - Modeling the Rear Suspension System of an Automobile

Introduction

In this project you will model your cars rear suspension system with a damped harmonic oscillator system of equations

$m*d^2y/dt^2 + k_d*dy/dt + k_s*y = 0$ where m is the mass of the system, k_d is the damping coefficient, and k_s is the spring coefficient.

In order to complete this model, you will need to find the values for m, k_d, and k_s. Once the model is complete, you will use the model to make predictions. Note: If you don't own a car, borrow a car of a friend - you don't have to drive the car, merely place your weight on the rear suspension. (This only takes a few minutes)

PROCEDURE

1. Find The Mass *m*.

The mass m may be calculated from the weight. The weight may be found at http://www.cars.com. Click on *Prices & Research* and you may find the "curb weight" of the car. Use 30% of this weight as an estimate - remember this is the rear end only.

The "mass" of the system is calculated by dividing the weight in pounds by 32 ft/sec^2 . For example, if the car weighs 2600 pounds, 30% of this is 780 pounds of FORCE.
The mass is 780/32 = 24.4 $(lb\text{-}sec^2)/ft$.
These mass units $lb\text{-}sec^2/ft$ seem unusual, but note that the product of "mass" and acceleration (ft/sec^2) results in pounds of force.

Record your mass (adjusted with factors of .30 and 32) and type of car.

2. Find The Spring Constant k_s

Recall from Hooke's Law of Springs: The restoring force exerted by a spring is directly proportional to the springs displacement from its rest position. In equation form this is $F = k_s*y$. In this model, a weight sufficient to displace the rear end of the car could be your body weight placed over the rear axle. Somehow find a way

to measure displacement (a friend is
helpful for this). Remember that we are using feet units, so convert inches to feet. Your weight (in pounds of force) is already in the correct units, so $k_s = F / y$ = (your weight)/displacement. Units of k_s are lb/ft. Record your value of k_s (did you use the correct units for y?).

3. Find The Damping Coefficient k_d

At this point, you are able to determine all of the parameters of the harmonic oscillator equation except k_d.

For example, you may obtain an equation of the form

$$24.2 * d^2y/dt^2 + k_d \, dy/dt + 2400\, y = 0$$

where m = 24.2 lb-sec^2/ft and k_s = 2400 lb/ft . (This is the equation I obtained with my car)

A simple way to approximate kd is to displace the auto 1 inch below the equilibrium level and then measure the time it take for the car to settle into the equilibrium position. Thus, you were starting with initial conditions t=0, y=-1/12 (feet), and y'=0.

Graph your equation on Graphmatica with different values of kd until the graph matches your observation. For the example above, I found that my car was close to "equilibrium" in a little less than ½ second. The value of k_d resulting in the closest matching graph was 400 lb-sec/ft. The form used to graph this example on Graphmatica is given below.

My equation was entered on Graphmatica as 24.2*d2y + 400dy +2400y = 0 {0,-1/12,0}

You must find your own value of k_d. Mine was 400.

Note that k_d = 400 and {0,-1/12,0} are the initial conditions, t=0, y=-1/12, y'=0. Time t corresponds to the x-axis in the Graphmatica plot.

You must determine your own value of k_d ! Record your data and observations

used to find k_d . Also, include any and all Graphmatica plots used to determine the value of k_d .
Write out your completed model.

4. Using Your Completed Model to Make Predictions

Once all of the parameters are determined, the effects of changes on the parameters such as increased mass or lack of damping (worn out shocks!) can be predicted. We will assume that ks remains constant.

A) Use your model and Graphmatica to predict what happens when the damping coefficient is cut in half. Write a paragraph explaining what happens. Discuss what actually could cause this dramatic decrease in damping in the real-life system. Include all graphs in your write-up.

B) Use your model and Graphmatica to predict what happens when the mass of the system is doubled. Write a paragraph explaining what happens. Discuss what actually could cause this dramatic increase in mass in the real-life system. Include all graphs in your write-up.