

PROJECT 4 ffee TIME - IS CALCULUS TIME!

In this project you will compare how well your favorite coffee cup keeps a hot cup of coffee hot. You will solve a differential equation for the temperature in the cup and use this to predict how long it will take for the coffee to get cold in the cup.

According to Newton's law of cooling and heating, the rate of cooling of an object is directly proportional to the difference of the temperature of the object and medium surrounding the object. This is given by the equation

$$\frac{dy}{dt} = k(y - M)$$

where dy/dt = rate of change of temperature of the substance,
 M = the temperature of the surrounding medium,
 y = the temperature of the substance at any given time t , and
 k = a constant that must be solved for.

Procedure

1. Record the room temperature. This is the temperature M from the above equation.
2. Find a coffee cup that you wish to test. Fill the cup with hot tap water (or coffee).
3. Place the thermometer into the cup. After a minute or two, record the temperature in the cup and record the time. Use a stopwatch if you have one. This is the initial temperature y_0 . (Time is $t = 0$)
4. Just before 6 minutes, stir the cup with the thermometer. Then measure the temperature of the cup at exactly 6 minutes.
5. Just before 20 minutes, stir the cup with the thermometer. Then measure the temperature of the cup at exactly 20 minutes.

ROOM TEMPERATURE = _____	
TIME	TEMPERATURE
INITIAL TIME ($t = 0$)	_____
6 MINUTES	_____
20 MINUTES	_____

WRITE-UP QUESTIONS ARE ON THE FOLLOWING PAGE

Write-up

1. Solve the separable equation $dy/dt = k(y - M)$ and obtain a general solution.
See Example 6 on P.362
2. Use the data at time $t=0$ and $t=6$ to find the particular solution for this experiment.
(Solve for k and C . $M = \text{room temp}$)
3. Compute the temperatures at time $t=20$ by using your particular solution that you just found.
4. Compute the percent difference between the measured temperature and computed temperature at $t=20$ minutes.

$$\text{This is } \frac{\text{Measured} - \text{Computed}}{\text{Measured}} \times 100\%$$

5. **Explain why the measured temperature was higher than the computed temperature.**
*Note the following as stated on P.362: **Newton's Law of Cooling** states that the rate of change in the temperature of an object is proportional to the difference between the object's temperature and the temperature of the *surrounding* medium.*
6. Using the particular solution, predict the time it will take for the temperature in the cup to be 1 degree higher than the room temperature.
7. Discuss ways in which your coffee cup could be made more efficient in holding in heat.