Absolute Value Equations

Equations with a variable or variables within absolute value bars are known as *Absolute Value Equations*.

Examples:

$|x - 3| = 5$

$|2x - 3| + x = 2$

Method To Solve Absolute Value Equations:
To solve absolute value equations, remember to do the following:

- Isolate the absolute value expression on one side of =. So for example, use the Addition Property of Equality to subtract x from both sides of $|2x - 3| + x = 2$ to result in $|2x - 3| = 2 - x$.

- Use the Absolute Value Equation Property to solve two cases without the absolute values, one positive and one negative. In the above example, you would solve **$2x - 3 = 2 - x$** and **$2x - 3 = -(2 – x)$.**

- After solving, *check all answers*. You may get extraneous solutions! In the example above, we would get answers of x=5/3 and x=1. It turns out that both work.

Example: Solve $|2x - 3| = 2$

First, apply the Absolute Value Equation Property to rewrite as two problems.
$2x - 3 = 2$ and **$2x - 3 = -2$**

Next, use the Addition Property of Equality to move all terms to one side in both equations, resulting in
$2x - 3 + 3 = 2 + 3$ and **$2x – 3 + 3 = -2 + 3$**
$2x = 5$ and **$2x = 1$**

Use the Division Property of Equality to solve both to get
$x = 5/2$ and **$x = 1/2$**.

Check both answers in $|2x - 3| = 2$
$|2(5/2) - 3| = 2$ and **$|2(1/2) - 3| = 2$ are both true!**

Example: Could you solve $|x^2 - 1| = x$ using this same method? Explain how.

Answer: Yes

First, apply the Absolute Value Equation Property to rewrite as two problems.
$x^2 - 1 = x$ and **$x^2 - 1 = - x$**

Next, use the Addition Property of Equality to move all terms to one side in both equations, resulting in

$x^2 - 1 + 1 = x + 1$ and $x^2 - 1 + 1 = -x + 1$

$x^2 = x + 1$ and $x^2 = 1 - x$

You now have two quadratic equations that can be rewritten as

$x^2 - x - 1 = 0$ and $x^2 + x - 1 = 0$ by applying the Addition Property of Equality.

You could then solve these using the Quadratic Formula as shown below. You would then have to check all four solutions!

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)} \longrightarrow x = \frac{1 \pm \sqrt{5}}{2}$$

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(-1)}}{2(1)} \longrightarrow x = \frac{-1 \pm \sqrt{5}}{2}$$

The easiest way to check these solutions is to use the decimal forms of the answers and plug them into $|x^2 - 1| = x$.