Graphing Functions Basics

Graphing Functions By Including Intercepts

A simple method of graphing only involves plotting points, chosen in more or less random manner. An improvement to this method involves plotting all intercepts first, and then plotting some additional points.

Intercepts Method For Graphing Functions

1. **Find and plot all intercepts**. To find y-intercepts, let x=0 and solve for y. To find x-intercepts, let y=0 and solve for x.
2. **Plot points on each side of each x-intercept**. Find and plot at least one point between each two x-intercepts and one point on each side of the largest and smallest x-intercept.
3. **Draw a smooth curve through the points** from left to right.

Example: Graph $f(x) = x^3 – x$.
First, rewrite as $y = x^3 – x$. Now, find intercepts.

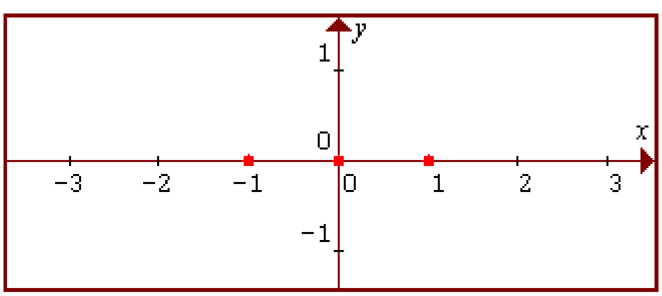
When $x = 0$, we get $y = 0^3 – 0 = 0$. So our y-intercept is (0,0)

When y=0, we get $0 = x^3 – x$. We solve this equation. CONTINUED ON NEXT PAGE

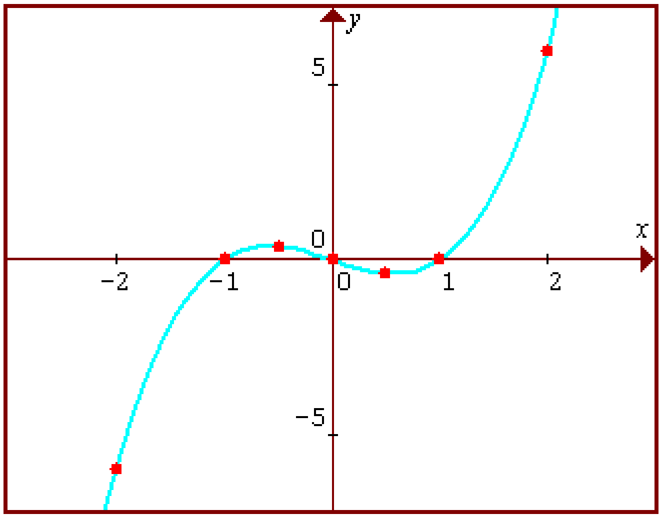
$0 = x^3 – x$	Given
$0 = x(x^2 – 1)$	Use Distributive Property to factor out x.
$0 = x(x + 1)(x – 1)$	Use Distributive Property to factor $x^2 – 1$.
$x = 0$	The Zero Product Law allows us to let each factor = 0
$x + 1 = 0$	
$x – 1 = 0$	
x = **-1**, x = **1**, x = **0**	Solve each equation using Addition Property of Equality

So the x-intercepts are (**-1**,0), (**1**,0), and (**0**,0)

We plot these points to get



Now, find and plot points on each side of each of these x-intercepts by letting x= -2, x=-1/2, x=1/2, and x=2. Draw a smooth curve through the points.



We get the points

x	y
-2	-6
2	6
-1/2	3/8
1/2	-3/8

Does This Intercepts Method ALWAYS Work?

No. It does work well for graphing factorable higher-order polynomials like $y = x^3 - x$, but when you start graphing rational functions like y = (x-2)/(x+1), or trigonometric functions like y = 2cos(x – 2), you will want to learn specialized methods that address the characteristics of these more advanced functions.

Example: Graph a 2-Part Function

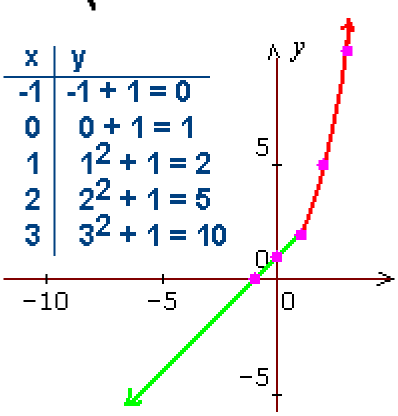
We may graph

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x \geq 1 \\ x + 1 & \text{if } x < 1 \end{cases}$$

by simply generating points for each part of the function using the rule determined by the value of x.

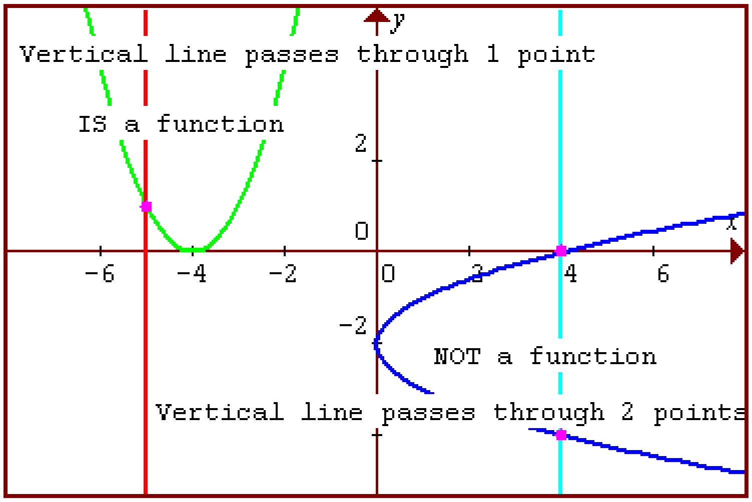
$$f(x) = \begin{cases} x^2 + 1 & \text{if } x \geq 1 \\ x + 1 & \text{if } x < 1 \end{cases}$$

x	y
-1	-1 + 1 = 0
0	0 + 1 = 1
1	$1^2 + 1 = 2$
2	$2^2 + 1 = 5$
3	$3^2 + 1 = 10$



For the part of the graph corresponding to y=x+1, we only need a few points since this the equation of a line. For the part of the graph corresponding to $y=x^2+1$, we get 3 points and it is clear that the graph is increasing in a vertical direction for larger and larger x-values.

The Vertical Line Test
If a formula represents y as a function of x, then each x may produce only one y output-value. In the graph, this means that there may not be two points at any given value of x, i.e. **if a vertical line passes through more than one point, then the graph does not represent y as a function of x.** The graphs below illustrate this.



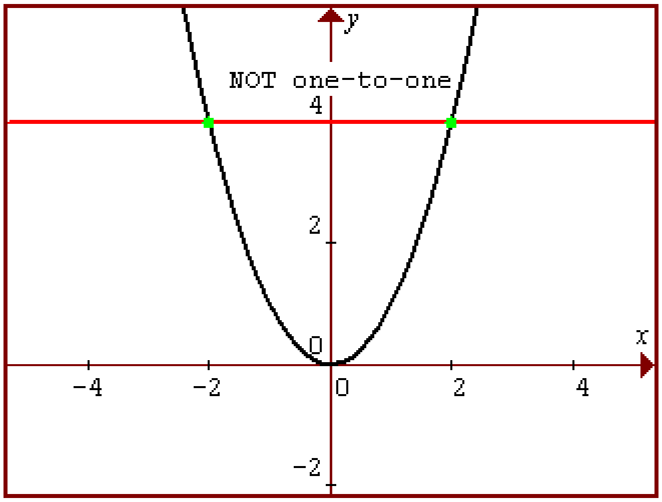
One-To-One Functions and The Horizontal Line Test
One-to-one functions are those that do not match two or more input values with the same output value.

Example: y = 4x + 1 is a one-to-one function because each y-value is produced by one and only x-value. For instance, if y=13, we can solve 13 = 4x + 1 for x to get a single solution, x=3.

Example: y = x^2 is not a one-to-one function since both x=2 and x=-2 result in y = 4.

How To Determine if a Function is One-To-One

- **Solve the equation for x** (or whatever the independent variable is). If you can not get a solution without a ± sign or absolute value sign, then the function is not one-to-one. For example, if we try to solve the function y = x^2 for x, we get x = ±√y. So f(x) = x^2 is not a one-to-one function, even though it still is a function.
- **Graph the equation**. If there are two x-values corresponding to a single y-value, then the function is *not* one-to-one. **In other words, if a horizontal line passes through two or more points, then the function is not one-to-one**. In the graph of y=x^2 below, there are two values of x, x=2 and x=-2, that result in y=4. This is known as **The Horizontal Line Test**. SEE NEXT PAGE FOR GRAPH



The Importance of One-To-One

If a function is one-to-one, we then may "work backwards" to recover the x-value used to produce a given y-value. For example, when encrypting messages, we perform operations (functions) on each character to produce new encrypted characters. We must be able to recover the original characters when we decode the message. So the function must be one-to-one so that we recover only one original message, and not a choice of many different messages. For an example on encryption that shows this, see http://www.mathmotivation.com/symbolic/secret-codes-lesson3.html .

Function Graph Definitions – Increasing, Decreasing, and Constant

A function is said to be **increasing** in a specified interval of x-values if an increase in x always results in an increase in the y-value within the specified interval of x-values.

A function is said to be **decreasing** in a specified interval of x-values if an increase in x always results in a decrease in the y-value within the specified interval of x-values.

A function is said to be **constant** in a specified interval of x-values if an increase in x always results in no change in the y-value within the specified interval of x-values.

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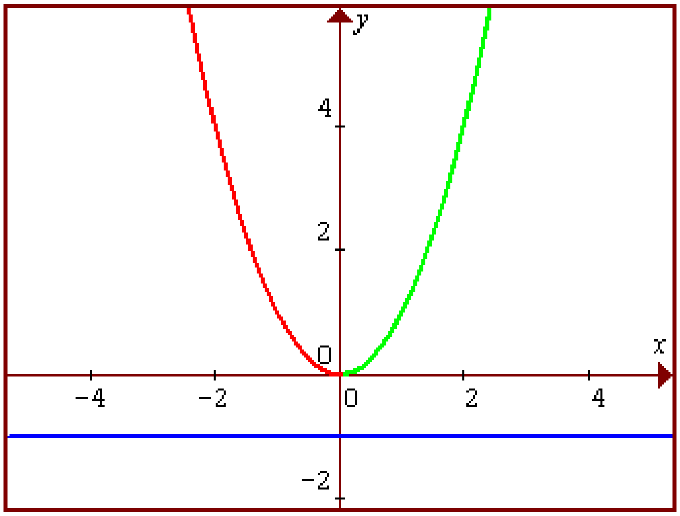
The graphs below illustrate these definitions for the functions $y=x^2$ and $y=-1$.



Here, the **function $y=x^2$ is increasing on the interval, $x > 0$**, which in interval notation is $(0,\infty)$. The right side of the graph slopes upward with positive slope in this interval.

The function **$y=x^2$ is decreasing on the interval $x < 0$**, which in interval notation is $(-\infty, 0)$. The left side of the graph slopes downward with negative slope in this interval.

The graph of $y=-1$ is a horizontal line. **$y = -1$ is constant for all real numbers**, i.e. the interval $(-\infty, \infty)$.