Introduction To Functions

A function from the set of x-values to the set of y-values is a rule or formula where x-values are input and the function assigns exactly 1 y-value to each x-value that is input.

The set of all x-values that are input is called the DOMAIN.

The set of all y-values that correspond to the given x-values is called the RANGE.

Examples of equations that represent y as a function of x, and some that don't:

y = 2x + 1 is a function of x since each x-value input results in only 1 y-value.

│y│=x is NOT a function of x since x=9 corresponds to both y = 9 *and* y = -9.

y = x^2 is a function of x since each x-value input results in only 1 y-value.

y^2 = x is NOT a function of x since x = 4 results in y = 2 *and* y = -2.

y = ±√x is NOT a function of x since x = 4 results in y = 2 *and* y = -2.

Function Notation
We may write a formula that defines a function with what is called "function notation". We replace y with f(x).

We call f(x) "f of x" and it means "the y=value when x is input."

Example: If f(x) = 2x + 1, what is f (3), f (- 5), and f(A)?

f(3) = 2(3) + 1 = 7
f(- 5) = 2(- 5) + 1 = - 9
f(A) = 2(A) + 1   or   2A + 1

To use Function Notation, just remember to replace each x in the original formula with whatever is after f within the parentheses!

Example: Given f(x) = x^2 + 1 , evaluate and simplify the expression below.

$$\frac{f(x+h) - f(x)}{h}$$

Here, you are probably wondering, "What is h?" We don't know what h is, so we simply carry it through all the operations as a variable.

Our first step in this problem is to evaluate the two expressions in the numerator. Then plug in these results and simplify. CONTINUED . . .

$$\frac{f(x+h) - f(x)}{h}$$

Given

$f(x + h) = (x + h)^2 + 1$
$= x^2 + xh + xh + h^2 + 1$
$= x^2 + 2xh + h^2 + 1$

Use the definition of function notation.

Use the Distributive Property, and combination of like terms to simplify.

$f(x) = x^2 + 1$
which is the given formula

Use the definition of function notation.

Now, substitute in the above results to get

$$\frac{x^2 + 2xh + h^2 + 1 - (x^2 + 1)}{h}$$

Apply the Distributive Property to add –1 times the second quantity, add like terms, and simplify to get

$$\frac{2xh + h^2}{h}$$

Apply the Distributive Property to factor, and then Cancel Factors of h to get

$$\frac{h(2x + h)}{h}$$

$= 2x + h$

The previous example illustrates a use of function notation used in Calculus. The quotient shown represents the derivative of f(x) when the value "h" is allowed to approach zero.

Notations Other Than f(x)
Besides f(x), a letter different than f may precede (x) in the notation. And in fact, variables other than x may, and are, often used. Here are some examples:

Examples:
g(x), h(x), and P(x) are often used instead of or in addition to f(x).
P(t) is often used to give the population P, as a function of time t.

So if $P(t) = 3t^2$ represented the population P as a function of t in years, then P(5) would equal $3 \bullet 5^2 = 75$, which would indicate that the population is 75 at t=5 years.

Two-Part Functions
Some functions are defined in two or more parts. For example, the function

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x \geq 1 \\ x + 1 & \text{if } x < 1 \end{cases}$$

uses the formula $y = x^2+1$ for x values ≥ 1 and uses the formula $y = x+1$ for x values < 1. So for example $f(4) = 4^2 + 1 = 16$ and $f(-1) = -1 + 1 = 0$.

Finding The Domain of a Function
The domain of a function consists of the largest set of input-values that results in a real output value of the function, unless otherwise restricted. With a function where y is a function of x, we are looking for the largest set of x-values that makes y a real number.

Example: Find the domain of f(x) = 1 / (x – 1).

In this case, we are finding the largest set of x-values that makes the y-value defined as $y = 1/(x-1)$ a real number.

Since x=1 results in division by zero, we know that x=1 is not part of the domain. All other real numbers result in a real y-value. Therefore **the domain of f(x) is All Real Numbers Except x = 1**.

Here are equivalent ways to state this domain **All Real Numbers Except x = 1** :
- (- ∞, 1) U (1, ∞)
- All Reals, x ≠ 1
- x ≠ 1

HINT: The way to find the domain of a function is to look for values of x that don't result in a real value of y. The domain is usually All Real Numbers Except For These Values. For example, in y= √x, all negative values of x will result in a non-real y-value. So the domain will be: All Reals *Except* Negative Numbers *or equivalently* All x ≥ 0.

Restricting a Domain For Certain Applications
In many applications, the input values for our function are restricted. For example, if we have a formula (function) that calculates the weight of a fish using it's length as an input value, the domain will not include negative values since a fish can not have a negative length. Or, if our input value consists of number of people visiting a shopping mall, we would restrict the domain to positive counting numbers since we can have neither negative numbers of people nor fractional numbers of people.